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## APPLICATION OF PROBABILISTIC-TIME GRAPHS FOR EVALUATING THE EFFECTIVENESS OF THE ELECTROCARDIOLOGICAL STUDY PROCESS

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### ABSTRACT

This work is devoted to the development of a structural model of the patient’s electrocardiological study process based on graph theory, probability theory and the method of generating functions. The developed structural model is presented in the form of a probabilistic-time graph, in which nine main states and an uncertainty state (a set of states that do not lead to the goal) are identified, as well as the probabilistic-time characteristics of the arcs of transitions from one graph state to another. The following are identified as the main states characterizing the process to complete an electrocardiological study: the beginning of the study; indications were defined; morphological analysis of biomedical signals with locally concentrated features was performed; pathological changes were identified; comparison with previous electrocardiological studies was performed; dynamics evaluation was completed; evaluation of treatment effectiveness was completed; diagnostic decision was made; recommendations were issued (the end of the electrocardiological study). For the proposed model of the electrocardiological study process by the Mason method, there are obtained analytical expressions for the generating functions of the entire graph, as well as the part of the graph that characterizes the successful completion of the electrocardiological study. Using the indicated generating functions, analytical expressions were obtained to calculate the average transit time of an electrocardiological study and the probability of successful completion of this process. To get all analytic expressions, a program was written in the Matlab language. The developed structural model of an electrocardiological study in the form of a probabilistic-time graph made it possible to identify the main states and determine the criteria for the effectiveness of the process in terms of average time and the probability of a successful study.

**Keywords:** Electrocardiological Study; Probabilistic-Time Graph; Generating Function; Mason Method; Biomedical Signals With Locally Concentrated Features

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### INTRODUCTION

Modern medicine is characterized by a sharp increase in the amount of information processed in solving traditional medical problems: from registering biomedical information to making a diagnosis, determining the prognosis, choosing and correcting treatment tactics based on the results of the diagnosis.

The principal advantage of the analysis of biomedical data using medical information systems is the possibility of simultaneous evaluation of many parameters with the processing of large amounts of information, which is beyond the power of either humans or automatic analyzers focused only on selected analysis methods [1, 2], [3].

One of the types of biomedical information, recorded in the form of curves, is biomedical signals (BMS) with locally concentrated features (LCF) associated with the cyclic work of the heart and cardiovascular system [4].

To automate the collection and processing of such information are the various computer cardiology systems.

### LITERATURE ANALYSIS AND PROBLEM STATEMENT

Traditionally, to diagnose the state of the cardiovascular system, morphological analysis of the electrocardiogram (ECG) is performed, in which the waves and complexes are distinguished [5, 6], their amplitude and time characteristics are determined, and the shape of the selected structural elements is also analyzed [7]. As a result of the morphological analysis, a set of diagnostic features is formed.

Most often, morphological analysis of BMS with LCF is performed in the time domain using modern classification methods, such as cluster analysis and pattern recognition [8, 9], probabilistic classification [10], neural networks [11], and fuzzy clustering [12,13]. At the same time, when implementing various classification methods, a number of general problems arise that are characteristic of ECG analysis, which include the following:

– manual signal marking is required, which is not possible for long ECG recordings (for example, Holter monitoring);

– a significant amount of training samples is required;

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– the process of generating a training sample is not trivial since it is based on the experience of the user (cardiologist), therefore it is error-prone and time-consuming;

– training methods on specific data sets can produce large errors on other sets due to the large variability of the ECG.

In addition, in the case of using cluster analysis, there are difficulties associated with the following factors:

- computational costs;
- the uncertainty of the number of classes and the initial partition;
- strong class asymmetry;
- signal variability, artifacts and so on.

Also, to solve the problem of the morphological analysis of BMS with LCF, methods of signal analysis in the time-frequency domain are used, for example, local (window) Fourier transform (spectral-time mapping) and wavelet transform [14, 15], as well as in the phase plane [4].

In addition, the authors proposed a method for morphological filtration of BMS with LCF, which also solves the problem of the morphological analysis of the ECG [16].

In addition, methods based on interpolation of the ECG isoelectric line by such methods as piecewise linear interpolation, splines and so on [22], as well as methods based on a combination of interpolation and isoelectric line filtering methods [23] are used to compensate of isoelectric line drift.

A literature review showed that a lot of attention is paid to the analysis of separate stages of the ECG study process, as well as their quality and effectiveness, however, a systematic analysis of the process as a whole is not performed.

On the other hand, there has recently been a tendency to use the mathematical apparatus of graph theory to study not the graphs themselves and their properties, but processes on them. That is, the graph is the basis on which effective models of the processes under consideration are built [24]. Of particular interest is the class of probabilistic-time graphs with which problems of analyzing processes in various application areas are solved, for example, analysis of message transmission in various computer and telecommunication networks [25,26], [27, 28], analysis of GERT networks [29], analysis of differential equations on graphs [30, 31], analysis of dynamic graph models and processes [32,33], analysis of knowledge representation of subject areas of computerized training systems [34], etc.

Therefore, the construction of a model of the ECG examination process to study the effectiveness

of the whole process is an actual scientific and applied task.

**The purpose of the article** is to increase the efficiency of the ECG study process by analyzing its probabilistic-time characteristics using graph theory methods in the design of cardiological decision support systems based on morphological analysis of BMS with LCF.

To achieve this goal, the following tasks are solved:

- to develop a structural model of the ECG study process using the mathematical apparatus of probabilistic-time graphs;
- to determine the probabilistic-time characteristics and criteria of the effectiveness of the ECG study process using the developed model.

## RESEARCH METHODS

To achieve the goals in the work, methods of theoretical analysis and modeling are used: graph theory to build a structural model of an ECG study; probability theory and the method of generating functions to assess the effectiveness of an ECG study without using and with using various cardiological decision support systems.

### DEVELOPMENT OF A STRUCTURAL MODEL OF A PATIENT'S ECG STUDY

Let us consider the main stages of the protocol formation of an ECG study by a cardiologist in order to highlight the key stages and the relationships between them to build a structural model  $M_s$  of the ECG study process.

For a complete and high-quality filling of the ECG study protocol by a cardiologist at the initial stage, it is necessary to indicate the reasons (indications) for the study, to briefly describe the previously performed instrumental studies (if any were), indicating the purpose of these studies (screening or to diagnose pathology after treatment), to briefly describe the medical history (anamnesis). This information is necessary for more effective decision making. In the presence of previous ECG studies, it is necessary to identify changes in the dynamics or determine the effectiveness of treatment.

The next stage is the detection of diagnostic features as a result of the morphological analysis of BMS with LCF. Here, depending on the type of ECG study, there are many standards that help to standardize the protocol of instrumental examination in the description of traditional diagnostic features. For example, a standard ECG protocol or a Holter monitoring protocol.

Based on the analysis of diagnostic features, the stage of detecting pathological changes is performed (for example, left ventricular hypertrophy, ventricular, atrial or nodal extrasystole, atrial fibrillation, etc.). The result of this stage is either a description of pathological changes or a statement of the fact that they are not detected.

Further, in the case of previous ECG studies, the results of the current and previous examinations are compared to assess the dynamics of changes if the control is carried out without treatment (for example, as a result of screening examinations) or to evaluate the effectiveness of treatment if treatment was performed between the examinations.

At the next stage, taking into account all the obtained results of ECG studies (both current and previous ones, if they were carried out), diagnostic decisions are made, which are made out in the form of an examination protocol.

At the final stage, the patient is given various recommendations on the tactics of further examination.

Let us imagine a structural model of an ECG study in the form of a probabilistic-time graph:

$$M_s = \langle S, T, P \rangle,$$

where:

$S = \{S_i\} \neq \emptyset$  – a set of states of the ECG study process ( $S_i$  is vertices of the graph);

$T = \{t_{ij}\} \neq \emptyset$  – a set of time characteristics of the transition from the state  $S_i$  to the state  $S_j$  of the ECG study process;

$P = \{p_{ij}\} \neq \emptyset$  – a set of probabilities of the transition from the state  $S_i$  to the state  $S_j$  of the ECG study process.

Let us define the arc from the state  $S_i$  to the state  $S_j$  by the vector of state indices  $(i, j)$  that this arc connects. Pairs  $(p_{ij}, t_{ij})$  describe the graph arcs  $(i, j)$  and determine the probability  $p_{ij}$  and time  $t_{ij}$  of transition from the state  $S_i$  to the state  $S_j$ .

To describe the progress of the ECG study process from the initial state to the final state, it is necessary to determine the arc function  $f(p_{ij}, t_{ij}) = f_{ij}(z)$  of a probabilistic-time graph such that when finding the products of the arc functions, the probabilities  $p_{ij}$  are multiplied and the times  $t_{ij}$  are summed:

$$f_{ij}(z) = p_{ij}z^{t_{ij}}, \tag{1}$$

where:  $z$  – a parameter of the arc function, the degree of which characterizes the time of transition from one state to another ( $|z| \leq 1$ ).

Moreover, for series and parallel connected arcs, the following expressions are valid:

$$f_s(z) = \prod_{(i,j)} f_{ij}(z) = \prod_{(i,j)} p_{ij}z^{t_{ij}}; \tag{2}$$

$$f_p(z) = \sum_{(i,j)} f_{ij}(z) = \sum_{(i,j)} p_{ij}z^{t_{ij}}, \tag{3}$$

where:

$f_s(z)$ ,  $f_p(z)$  – the functions of series and parallel connected arcs, respectively.

If the graph contains a vertex that has a loop with the function  $f_{ii}(z)$  and an arc with the function  $f_{ij}(z)$ , then both arcs are replaced by one with the function  $f_{ij}^e(z)$  calculated by the expression

$$f_{ij}^e(z) = \frac{f_{ij}(z)}{1 - f_{ii}(z)}. \tag{4}$$

Then the generating function  $F(z)$  corresponding to the probabilistic-time graph describing the ECG study process is calculated as the sum of the functions of all parallel paths from the initial state to the final ones.

To calculate the generating function  $F(z)$ , the method of equivalent transformations is used, the purpose of which is to obtain the simplest transition graph from the initial to the final state [35, 36].

For example, let us consider the graph consisting of three vertices and having one feedback (Fig. 1a). Let us consider the transition from the state  $S_0$  to the state  $S_2$ . The stages of equivalent conversion are shown in Fig. 1b, c.

Taking into account rules (2) and (4), there is obtain the following expression for calculating the equivalent function  $f_{02}^e(z)$  of the arc  $(0, 2)$ :

$$f_{02}^e(z) = \frac{f_{01}(z)f_{12}(z)}{1 - f_{01}(z)f_{10}(z)}.$$

If the graph has a complex structure then it is not always possible to bring the graph to the simplest form by the method of equivalent transformations (or such transformations will be too time-consuming). In this case, the Mason method [37] is more efficient, according to which the generating function  $F_{0i}(z)$  between the initial state  $S_0$  and any state  $S_i$  can be calculated using the following expression:

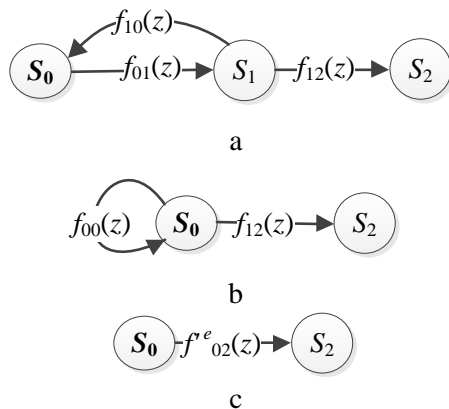
$$F_{0i}(z) = \frac{\sum_{k=1}^m f_k(z)\Delta_k}{\Delta}, \quad (5)$$

where:

$f_k(z)$  – the function of the  $k$ -th simple (open) path from the initial state  $S_0$  to the state  $S_i$  calculated by (2);

$\Delta_k$  – the determinant of that part of the graph that does not come into touch with the  $k$ -th simple path (i.e., with the part of the graph that remains after exclusion of contours touching with the  $k$ -th path);

$\Delta$  – the graph determinant.



**Fig. 1. Graph example: a) original graph; b) equivalent transformation of parallel arcs; c) equivalent transformation of loop-arc**

Source: compiled by the author

The determinant is calculated by the following expression:

$$\Delta = 1 - \sum_j f_{(c)j}(z) + \sum_{j,k} f_{(c)j}(z)f_{(c)k}(z) - \sum_{j,k,l} f_{(c)j}(z)f_{(c)k}(z)f_{(c)l}(z) + \dots \quad (6)$$

where:

$\sum_j f_{(c)j}(z)$  – the sum of the functions  $f_{(c)j}(z)$

of all contours;

$\sum_{j,k} f_{(c)j}(z)f_{(c)k}(z)$  – the sum of the products of

the functions  $f_{(c)j}(z)$  and  $f_{(c)k}(z)$  of not touching pairs of contours;

$\sum_{j,k,l} f_{(c)j}(z)f_{(c)k}(z)f_{(c)l}(z)$  – the sum of the

products of the functions  $f_{(c)j}(z)$ ,  $f_{(c)k}(z)$  and  $f_{(c)l}(z)$  of not touching triples of contours, etc.

According to the theory of generating functions and its application to probability theory, the expected value of a sequence of random variables (which in this case are  $t_{ij}$ ) can be expressed through the generating function of the sequence as the value of the first derivative at  $z=1$ . Thus, the probability  $P_{ECG}$  and average time  $T_{ECG}$  to complete an ECG study are determined with the found generating function using the following expressions [35]:

$$P_{ECG} = F(z)|_{z=1}; \quad (7)$$

$$T_{ECG} = \left. \frac{dF(z)}{dz} \right|_{z=1}. \quad (8)$$

Based on the above stages of the ECG study process, 9 main states can be identified:

$S_0$  – the beginning of the study;

$S_1$  – indications were defined;

$S_2$  – morphological analysis of BMS with LCF was performed;

$S_3$  – pathological changes were identified;

$S_4$  – comparison with previous ECG studies was performed;

$S_5$  – dynamics evaluation was completed;

$S_6$  – evaluation of treatment effectiveness was completed;

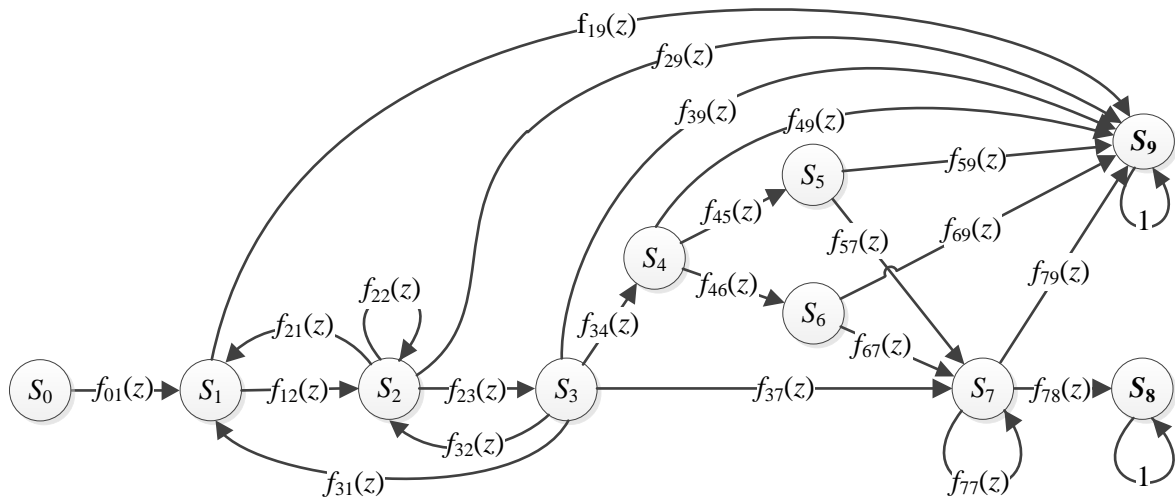
$S_7$  – the diagnostic decision was made;

$S_8$  – recommendations were issued (the end of the ECG study).

Let us define one more state  $S_9$  – the state of uncertainty into which the ECG study process can go from the main states if for one reason or another it is impossible to transit from one main state to another. There can be several reasons for the transition to a state of uncertainty, while the reasons can be both objective and subjective. Objective reasons include equipment failure, power outages, poor signal quality due to various types of interference (for example, poor contact of one or more electrodes), lack of a cardiologist (for example, due to illness), etc. As subjective reasons, one can consider the patient’s reluctance to undergo the ECG study, for example, due to financial difficulties or due to a psychoemotional state. In addition, errors that occur at each stage lead the system to the state of uncertainty.

Let us also set the probabilities and time of transition from one state to another.

Then, as a structural model  $M_S$  of the ECG research process, a probabilistic-time graph is proposed that reflects the main states of this process and their interaction (Fig. 2).



**Fig. 2. The structural model  $M_s$  of the ECG study:  $S_0$  – the beginning of the study;  $S_1$  – indications were defined;  $S_2$  – morphological analysis of BMS with LCF was performed;  $S_3$  – pathological changes were identified;  $S_4$  – comparison with previous ECG studies was performed;  $S_5$  – dynamics evaluation was completed;  $S_6$  – evaluation of treatment effectiveness was completed;  $S_7$  – the diagnostic decision was made;  $S_8$  – recommendations were issued (the end of the ECG study);  $S_9$  – a set of states that do not lead to the goal (the state of uncertainty);  $f_{ij}(z), \forall i, j = \overline{0;9}$  – arc function by (1)**  
 Source: compiled by the author

It should be noted that the structural model  $M_s$  is no state associated with the direct recording of the ECG signal. This is because the duration of the ECG signal recording is strictly regulated by the protocol of one or another type of ECG study and can vary from several minutes (in the case of a standard ECG study) to several hours and days (in the case of Holter monitoring). Because this time cannot be optimized, and the duration of the recording process does not affect the effectiveness of the ECG study, then this state is not taken into account in the structural model  $M_s$ .

If there is no arc  $(i, j)$  in the graph (Fig. 2), then the transition probability  $p_{ij} = 0$ , and therefore, according to (1) also  $f_{ij}(z) = 0$ .

One of the most time-consuming and crucial stages is morphological analysis of BMS with LCF, therefore, in case of recording a signal of poor quality, it is possible to return from the state  $S_2$  to the state  $S_1$  for re-recording the signal (the arc with the function  $f_{21}(z)$  in Fig. 2), and if necessary, correction of errors of the morphological analysis is provided the possibility of repeating the refinement of the morphological analysis of BMS with LCF (the loop with the function  $f_{22}(z)$  in Fig. 2).

The next stage of the ECG study, as noted above, is the stage of identifying pathological changes, which is based on the results of the morphological analysis. If this stage cannot be completed due to errors in the morphological analysis, then a return to the previous stage is possible, i.e. transition from the state  $S_3$  to  $S_2$  (the arc with the function  $f_{32}(z)$  in Fig. 2), and if due to a poor-quality signal, it is possible to transition from  $S_3$  to  $S_1$  (the arc with the function  $f_{31}(z)$  in Fig. 2).

And, finally, it is possible to repeat the stage of making a diagnostic decision (the loop with the function  $f_{77}(z)$  in Fig. 2), if there are difficulties in making a decision (for example, a consultation of other doctors is necessary).

Since all transitions from the current state  $S_i$  form a complete group of events then it is possible write the following expression:

$$\sum_j p_{ij} = 1. \tag{9}$$

Then, taking into account (9), we have the following expressions for the probabilities of a transition from any state  $S_i$  ( $i \in \{1,8\}$ ) to the state of

uncertainty  $S_9$  (i.e., an unsuccessful completion of an ECG study):

$$\begin{aligned}
 p_{19} &= 1 - p_{12}; \\
 p_{29} &= 1 - p_{23} - p_{21} - p_{22}; \\
 p_{39} &= 1 - p_{34} - p_{37} - p_{31} - p_{32}; \\
 p_{49} &= 1 - p_{45} - p_{46}; \\
 p_{59} &= 1 - p_{57}; \\
 p_{69} &= 1 - p_{67}; \\
 p_{79} &= 1 - p_{78} - p_{77}.
 \end{aligned}$$

To calculate the generating function of the proposed probabilistic-time graph, it is necessary to perform a series of equivalent transformations taking into account expressions (2)-(4) or use the Mason method (5).

**DEFINITION OF THE PROBABILISTIC-TIME CHARACTERISTICS OF THE STRUCTURAL MODEL OF A PATIENT'S ECG STUDY**

It is easy to notice that the presented graph has sequential and parallel branches, arcs in the form of loops, and also feedbacks. Since it is rather difficult to perform equivalent transformations for the graph under consideration, let us use the Mason method to calculate the generating function of this graph.

First, using rule (2), there is define the functions of all direct paths from  $S_0$  to  $S_8$  (there are three such paths in total):

$$\begin{aligned}
 f_{08}^1(z) &= f_{01}(z)f_{12}(z)f_{23}(z)f_{37}(z)f_{78}(z); \\
 f_{08}^2(z) &= f_{01}(z)f_{12}(z)f_{23}(z)f_{34}(z)f_{45}(z)f_{57}(z)f_{78}(z); \\
 f_{08}^3(z) &= f_{01}(z)f_{12}(z)f_{23}(z)f_{34}(z)f_{46}(z)f_{67}(z)f_{78}(z).
 \end{aligned}$$

Then, according to rule (3), the path function from  $S_0$  to  $S_8$  is determined as follows:

$$f_{08}(z) = f_{08}^1(z) + f_{08}^2(z) + f_{08}^3(z) = \sum_{k=1}^3 f_{08}^k(z). \tag{10}$$

Further, using rule (2), there is define the functions of all direct paths from  $S_0$  to  $S_9$  (there are seven such paths in total):

$$\begin{aligned}
 f_{09}^1(z) &= f_{01}(z)f_{19}(z); \\
 f_{09}^2(z) &= f_{01}(z)f_{12}(z)f_{29}(z);
 \end{aligned}$$

$$\begin{aligned}
 f_{09}^3(z) &= f_{01}(z)f_{12}(z)f_{23}(z)f_{39}(z); \\
 f_{09}^4(z) &= f_{01}(z)f_{12}(z)f_{23}(z)f_{34}(z)f_{49}(z); \\
 f_{09}^5(z) &= f_{01}(z)f_{12}(z)f_{23}(z)f_{34}(z)f_{45}(z)f_{59}(z); \\
 f_{09}^6(z) &= f_{01}(z)f_{12}(z)f_{23}(z)f_{34}(z)f_{46}(z)f_{69}(z); \\
 f_{09}^7(z) &= f_{01}(z)f_{12}(z)f_{23}(z)f_{37}(z)f_{79}(z); \\
 f_{09}^8(z) &= f_{01}(z)f_{12}(z)f_{23}(z)f_{34}(z)f_{45}(z)f_{57}(z)f_{79}(z); \\
 f_{09}^9(z) &= f_{01}(z)f_{12}(z)f_{23}(z)f_{34}(z)f_{46}(z)f_{67}(z)f_{79}(z).
 \end{aligned}$$

Then, according to rule (3), the path function from  $S_0$  to  $S_9$  is determined as follows:

$$f_{09}(z) = \sum_{k=1}^9 f_{09}^k(z). \tag{11}$$

Now, using rule (2), there is define the functions of all possible contours  $L_i$  ( $i = \overline{1,6}$ ):

$$f_{L_1} = f_{12}(z)f_{21}(z); \tag{12}$$

$$f_{L_2} = f_{22}(z); \tag{13}$$

$$f_{L_3} = f_{23}(z)f_{32}(z); \tag{14}$$

$$f_{L_4} = f_{12}(z)f_{23}(z)f_{31}(z); \tag{15}$$

$$f_{L_5} = f_{77}(z); \tag{16}$$

$$f_{L_6} = f_{12}(z)f_{23}(z)f_{32}(z)f_{21}(z). \tag{17}$$

Then, taking into account expressions (10)-(17), using the Mason rule (5), (6), it is obtain the following generating functions  $F_{08}(z)$  for the transition from  $S_0$  to  $S_8$  and  $F_{09}(z)$  for the transition from  $S_0$  to  $S_9$ :

$$F_{08}(z) = \frac{f_{08}(z)}{1 - \sum_{i=1}^6 f_{L_i}(z) + f_{L_5}(z) \sum_{i=1,2,3,4,6} f_{L_i}(z)};$$

$$F_{09}(z) = \frac{f_{09}(z)}{1 - \sum_{i=1}^6 f_{L_i}(z) + f_{L_5}(z) \sum_{i=1,2,3,4,6} f_{L_i}(z)}.$$

As a result, according to rule (3), the generating function of the graph shown in Fig. 2 is as follows:

$$F(z) = F_{08}(z) + F_{09}(z).$$

Analytical expressions for calculating the average time to complete an ECG study, as well as the likelihood of a successful ECG study, can allow exploring the effectiveness of the process under various initial conditions. Therefore, to find the analytical expression of the generating function, the probability  $P_{ECG}$  by (7), the average time  $T_{ECG}$  by (8) to complete an ECG study, and also the probability  $P_{ECG}^+$  of a successful ECG study, a program was written in the Matlab language, which is presented below:

```

% variables
syms p01 p12 p23 p34 p45 p46 p57 p67;
syms p37 p78 p21 p22 p31 p32 p77;
syms p19 p29 p39 p49 p59 p69 p79 ;
syms t01 t12 t23 t34 t45 t46 t57 t67;
syms t37 t78 t21 t22 t31 t32 t77;
syms t19 t29 t39 t49 t59 t69 t79;
syms z;

% straight arcs
f01(z)=p01*z^t01;
f12(z)=p12*z^t12;
f23(z)=p23*z^t23;
f34(z)=p34*z^t34;
f45(z)=p45*z^t45;
f57(z)=p57*z^t57;
f46(z)=p46*z^t46;
f67(z)=p67*z^t67;
f37(z)=p37*z^t37;
f78(z)=p78*z^t78;

% to the state S9
f19(z)=(1-p12)*z^t19;
f29(z)=(1-p23-p21-p22)*z^t29;
f39(z)=(1-p34-p37-p31-p32)*z^t39;
f49(z)=(1-p45-p46)*z^t49;
f59(z)=(1-p57)*z^t59;
f69(z)=(1-p67)*z^t69;
f79(z)=(1-p78-p77)*z^t79;

% back arcs
f21(z)=p21*z^t21;
f22(z)=p22*z^t22;
f31(z)=p31*z^t31;
f32(z)=p32*z^t32;
f77(z)=p77*z^t77;

% the direct paths from S0 to S8
f1_08(z)=f01(z)*f12(z)*f23(z)...
*f37(z)*f78(z);
f2_08(z)=f01(z)*f12(z)*f23(z)...
*f34(z)*f45(z)*f57(z)*f78(z);
f3_08(z)=f01(z)*f12(z)*f23(z)...
*f34(z)*f46(z)*f67(z)*f78(z);

% total direct path from S0 to S8
f08(z)=f1_08(z)+f2_08(z)+f3_08(z);

% the direct paths from S0 to S9
f1_09(z)=f01(z)*f19(z);
f2_09(z)=f01(z)*f12(z)*f29;
f3_09(z)=f01(z)*f12(z)*f23(z)*f39(z);
f4_09(z)=f01(z)*f12(z)*f23(z)...
*f34(z)*f49(z);

f5_09(z)=f01(z)*f12(z)*f23(z)...
*f34(z)*f45(z)*f59(z);

f6_09(z)=f01(z)*f12(z)*f23(z)*...
f34(z)*f46(z)*f69(z);

f7_09(z)=f01(z)*f12(z)*f23(z)*...
f37(z)*f79(z);
f8_09(z)=f01(z)*f12(z)*f23(z)...
*f34(z)*f45(z)*f57(z)*f79(z);

f9_09(z)=f01(z)*f12(z)*f23(z)...
*f34(z)*f46(z)*f67(z)*f79(z);

% total direct path from S0 to S9
f09(z)=f1_09(z)+f2_09(z)+f3_09(z)+ ...
f4_09(z)+f5_09(z)+f6_09(z)+ ...
f7_09(z)+f8_09(z)+f9_09(z);

% contours
L1(z)=f12(z)*f21(z);
L2(z)=f22(z);
L3(z)=f23(z)*f32(z);
L4(z)=f12(z)*f23(z)*f31(z);
L5(z)=f77(z);
L6(z)=f12(z)*f23(z)*f32(z)*f21(z);

% calculation of analytical
expressions
diary ('myVariable.txt');

% generating function of the path from
% S0 to S8
F08(z)=simplifyFraction(f08(z)/...
(1-(L1(z)+L2(z)+L3(z)+L4(z)+...
L5(z)+L6(z))+L5(z)*(L1(z)+...
L2(z)+L3(z)+L4(z)+L6(z))))

% generating function of the path from
% S0 to S9
F09(z)=simplifyFraction(f09(z)/...
(1-(L1(z)+L2(z)+L3(z)+L4(z)+...
L5(z)+L6(z))+L5(z)*(L1(z)+...
L2(z)+L3(z)+L4(z)+L6(z))))

% generating function of the graph
F(z)=simplifyFraction(F08(z)+F09(z))
% probability of performing
% an ECG study
P=(F(1))
T1(z)=simplifyFraction(diff(F(z)))

% average time of performing
% an ECG study
T=simplifyFraction(T1(1))

```

diary off

from  $S_0$  to  $S_8$  and  $S_9$ , are obtained, presented below:

As a result of the program execution, analytical expressions of the generating functions of the paths

$$F_{08}(z) = \frac{P_{01}P_{12}P_{23}P_{78} \left( P_{34}z^{t_{34}} \left( P_{45}P_{57}z^{t_{45}+t_{57}} + P_{46}P_{67}z^{t_{46}+t_{67}} \right) + P_{37}z^{t_{37}} \right) z^{t_{01}+t_{12}+t_{23}+t_{78}}}{(1-p_{77}z^{t_{77}}) \left( 1-p_{12} \left( p_{21} \left( 1+p_{23}P_{32}z^{t_{23}+t_{32}} \right) z^{t_{21}} + p_{23}P_{31}z^{t_{23}+t_{31}} \right) z^{t_{12}} - p_{23}P_{32}z^{t_{23}+t_{32}} - p_{22}z^{t_{22}} \right)}; \quad (18)$$

$$F_{09}(z) = p_{01} \left( (1-p_{12})z^{t_{19}} + p_{12} \left( (1-p_{21}-p_{22}-p_{23})z^{t_{29}} + p_{23} \left( (1-p_{31}-p_{32}-p_{34}-p_{37})z^{t_{39}} + p_{34} \left( (1-p_{45}-p_{46})z^{t_{49}} + p_{45} \left( (1-p_{57})z^{t_{59}} + p_{57} \left( 1-p_{77}-p_{78} \right) z^{t_{57}+t_{79}} \right) z^{t_{45}} + p_{46} \left( (1-p_{67})z^{t_{69}} + p_{67} \left( 1-p_{77}-p_{78} \right) z^{t_{67}+t_{79}} \right) z^{t_{46}} \right) z^{t_{34}} + p_{37} \left( 1-p_{77}-p_{78} \right) z^{t_{37}+t_{79}} \right) z^{t_{23}} \right) z^{t_{12}} \right) z^{t_{01}} / \left( (1-p_{77}z^{t_{77}}) \left( 1-p_{12} \left( p_{21} \left( 1+p_{23}P_{32}z^{t_{23}+t_{32}} \right) z^{t_{21}} + p_{23}P_{31}z^{t_{23}+t_{31}} \right) z^{t_{12}} - p_{23}P_{32}z^{t_{23}+t_{32}} - p_{22}z^{t_{22}} \right) \right).$$

Since the analytical expressions for the generating function  $F(z)$ , probability  $P_{ECG}$  and average time  $T_{ECG}$  of the ECG study are too cumbersome, they are not given in this article, although they can be easily obtained using the above

program, in which the variable  $F - F(z)$ , the variable  $P - P_{ECG}$ , and the variable  $T - T_{ECG}$ .

According to expression (7), calculating function (18) at  $z = 1$ , there are obtain the following expression for calculating the probability of a successful ECG study:

$$P_{ECG}^+ = F_{08}(z)|_{z=1} = \frac{P_{01}P_{12}P_{23}P_{78} \left( P_{34} \left( P_{45}P_{57} + P_{46}P_{67} \right) + P_{37} \right)}{(1-p_{77}) \left( 1-p_{12} \left( p_{21} \left( 1+p_{23}P_{32} \right) + p_{23}P_{31} \right) - p_{23}P_{32} - p_{22} \right)}. \quad (19)$$

If the ECG study does not provide for the implementation of one of the stages (for example, the patient has not have an ECG study before or has not have preliminary treatment), then we will accept the probability of transition to the corresponding state  $p_{ij} = 0$ , and therefore, according to (1) also  $f_{ij}(z) = 0$ .

For the proposed structural model of an ECG study, such conditions can be accepted for the vertices  $S_4$ ,  $S_5$  and  $S_6$ .

Based on the logic of conducting an ECG study, both the dynamics evaluation and the treatment effectiveness evaluation are not performed simultaneously in one study, i.e. the ECG study process is only one of the possible paths ( $p_{45} = 0$  or  $p_{46} = 0$ ).

Therefore, the following restriction must be imposed on the probabilities of transitions from the state  $S_4$  to the states  $S_5$  and  $S_6$ :

$$\begin{cases} p_{45}p_{46} = 0; \\ p_{45} + p_{46} \in (0;1]. \end{cases} \quad (20)$$

In addition, if previous ECG studies have not been conducted, then  $p_{34} = 0$ , otherwise  $p_{37} = 0$ , i.e. the following restrictions can be written:

$$\begin{cases} p_{34}p_{37} = 0; \\ p_{34} + p_{37} \in (0;1]. \end{cases} \quad (21)$$

Thus, according to the structural model  $M_s$  of the ECG study (Fig. 1), there are three alternative paths of transition from the initial state  $S_0$  to the final state  $S_8$ , which correspond to three different types of ECG studies:

- 1) the study is being conducted for the first time ( $p_{34} = 0$ , values  $p_{45}$  and  $p_{46}$  not important);
- 2) the study is repeated as a result of screening ( $p_{37} = 0$  and  $p_{46} = 0$ );
- 3) the study is conducted after treatment ( $p_{37} = 0$  and  $p_{45} = 0$ ).

Combining expression (19) and restrictions (20), (21), there is obtain the following system that allows to calculate the probability of a successful ECG study:



$$\begin{cases} P_{ECG}^+ = \frac{p_{01}p_{12}p_{23}p_{78}(p_{34}(p_{45}p_{57} + p_{46}p_{67}) + p_{37})}{(1 - p_{77})(1 - p_{12}(p_{21}(1 + p_{23}p_{32}) + p_{23}p_{31}) - p_{23}p_{32} - p_{22})}; \\ p_{34}p_{37} = 0; \\ p_{34} + p_{37} \in (0;1]; \\ p_{45}p_{46} = 0; \\ p_{45} + p_{46} \in (0;1]. \end{cases} \quad (22)$$

Obviously, restrictions (20) and (21) are also valid for calculating the average time of an ECG study.

The obtained analytical expressions of the average time  $T_{ECG}$  of an ECG study (the article provides a program for obtaining this expression) and the probability  $P_{ECG}^+$  of a successful ECG study (22) allow to formulate the following criteria for the effectiveness of the process under consideration:

$$\begin{cases} T_{ECG} \rightarrow \min; \\ p_{34}p_{37} = 0; \\ p_{34} + p_{37} \in (0;1]; \\ p_{45}p_{46} = 0; \\ p_{45} + p_{46} \in (0;1]. \end{cases}$$

$$\begin{cases} P_{ECG}^+ \rightarrow \max; \\ p_{34}p_{37} = 0; \\ p_{34} + p_{37} \in (0;1]; \\ p_{45}p_{46} = 0; \\ p_{45} + p_{46} \in (0;1]. \end{cases}$$

Thus, the developed structural model of the ECG study in the form of a probabilistic-time graph made it possible to obtain analytical expressions that describe the process under given initial conditions (the presence or absence of previous studies and treatment), as well as to determine the criteria for the

effectiveness of the ECG study, which can be further use for analysis and optimization of both the entire process and its individual stages.

## CONCLUSION

In the work, the structural model of the ECG study process using the apparatus of probabilistic-time graphs is developed. To build this model, 9 main states of the process under consideration are identified, as well as the main stages that significantly affect the effectiveness of the ECG study.

For the first time, an analytical expression is obtained that reflects the dependence of the average time of an ECG study on the time it takes for each stage to pass and the probabilities of transition from one state to another, as well as an analytical expression reflecting the dependence of the probability of a successful ECG study on the probabilities of successful completion of each stage.

The proposed model made it possible to formulate criteria for the effectiveness of the ECG study process by the average time of the study and the probability of its successful completion.

Further studies are aimed at experimental verification of the effectiveness of ECG studies without and with the use of various cardiological decision support systems using the obtained analytical expressions of the probabilistic-time characteristics of the developed model.

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## ЗАСТОСУВАННЯ ІМОВІРНІСНО-ЧАСОВИХ ГРАФІВ ДЛЯ ОЦІНКИ ЕФЕКТИВНОСТІ ПРОЦЕСУ ЕЛЕКТРОКАРДИОЛОГІЧНОГО ДОСЛІДЖЕННЯ

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### АНОТАЦІЯ

Дана робота присвячена розробці структурної моделі процесу електрокардіологічного дослідження пацієнта на основі теорії графів, теорії ймовірностей і методу твірних функцій. Розроблена структурна модель представлена у вигляді ймовірнісно-часового графа, в якому виділені дев'ять основних станів і стан невизначеності (безліч станів, які не ведуть до мети), а також визначені ймовірнісно-часові характеристики дуг переходів з одного стану графа в інший. В якості основних станів, що характеризують процес проведення електрокардіологічного дослідження, виділено такі: початок дослідження; визначені показання; виконаний морфологічний аналіз біомедичних сигналів з локально зосередженими ознаками; визначені патологічні зміни; виконано порівняння з попередніми електрокардіологічними дослідженнями; виконано оцінку динаміки; виконано оцінку ефективності лікування; прийнято діагностичне рішення; видані рекомендації (кінець електрокардіологічного дослідження). Для запропонованої моделі процесу електрокардіологічного дослідження методом Мезона отримано аналітичні вирази твірних функцій всього графа, а також частини графа, що характеризує успішне проходження електрокардіологічного дослідження. За допомогою зазначених твірних функцій отримано аналітичні вирази для розрахунку середнього часу проходження електрокардіологічного дослідження і ймовірності успішного завершення даного процесу. Для отримання всіх аналітичних виразів була написана програма на мові Matlab. Розроблена структурна модель електрокардіологічного дослідження у вигляді ймовірнісно-часового графа дозволила виділити основні стани та визначити критерії ефективності проведення зазначеного процесу по середньому часу і ймовірності успішного проходження дослідження.

**Ключові слова:** електрокардіологічне дослідження; ймовірнісно-часовий граф; твірна функція; метод Мезона; біомедичний сигнал з локально-зосередженими ознаками

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## ПРИМЕНЕНИЕ ВЕРОЯТНОСТНО-ВРЕМЕННЫХ ГРАФОВ ДЛЯ ОЦЕНКИ ЭФФЕКТИВНОСТИ ПРОЦЕССА ЭЛЕКТРОКАРДИОЛОГИЧЕСКОГО ИССЛЕДОВАНИЯ

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### АННОТАЦИЯ

Данная работа посвящена разработке структурной модели процесса электрокардиологического исследования пациента на основе теории графов, теории вероятностей и метода производящих функций. Разработанная структурная модель представлена в виде вероятностно-временного графа, в котором выделены девять основных состояний и состояние неопределенности (множество состояний, не ведущих к цели), а также определены вероятностно-временные характеристики дуг переходов из одного состояния графа в другое. В качестве основных состояний, характеризующих

процесс проведения электрокардиологического исследования, выделены следующие: начало исследования; определены показания; выполнен морфологический анализ биомедицинских сигналов с локально сосредоточенными признаками; определены патологические изменения; выполнено сравнение с предыдущими электрокардиологическими исследованиями; выполнена оценка динамики; выполнена оценка эффективности лечения; принято диагностическое решение; выданы рекомендации (конец электрокардиологического исследования). Для предложенной модели процесса электрокардиологического исследования методом Мэсона получены аналитические выражения производящих функций всего графа, а также части графа, характеризующей успешное прохождение электрокардиологического исследования. С помощью указанных производящих функций получены аналитические выражения для расчета среднего времени прохождения электрокардиологического исследования и вероятности успешного завершения рассматриваемого процесса. Для получения всех аналитических выражений была написана программа на языке Matlab. Разработанная структурная модель электрокардиологического исследования в виде вероятностно-временного графа позволила выделить основные состояния и определить критерии эффективности проведения указанного процесса по среднему времени и вероятности успешного прохождения исследования.

**Ключевые слова:** электрокардиологическое исследование; вероятностно-временной граф; производящая функция; метод Мэсона; биомедицинский сигнал с локально-сосредоточенными признаками

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