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Evaluation of the accuracy of human eye movement system identification using step test signals

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ABSTRACT

For mathematical modeling of the human eye movement system (EMS), integral nonlinear models are employed, which simultaneously account for the nonlinear dynamics and inertial properties of the research object. Based on experimental "input-output" studies of the EMS, diagonal cross-sections of multidimensional transient characteristics (MTCs) of second and third orders are determined. Experimental data are obtained using innovative eye-tracking technology, which allows the registration of ocular responses to visual test stimuli. The aim of this study is to investigate the accuracy of EMS identification based on eye-tracking data by evaluating errors in MTC computation using three nonlinear dynamic identification methods: approximation, compensation, and the least squares method (LSM), based on models in the form of integro-power series (IPS) and integro-power polynomials (IPP). The research object is the process of nonparametric EMS identification using Volterra models in the time domain. The research subject involves computational and software tools for determining EMS dynamic characteristics using eye-tracking data and analyzing the accuracy of models obtained with the specified methods. Accuracy evaluations were conducted for various EMS models (linear, quadratic, and cubic) based on three responses to test signals of varying amplitudes. For the approximation method and LSM, identical models were obtained when using the same test signals, as the models converge within the IPS convergence domain. The compensation method requires minimal computational resources compared to other methods; however, the models obtained using this approach exhibit significant errors, rendering them unsuitable for diagnostic studies. Third-order models showed instability in the estimates of transient characteristics. Error analysis of EMS dynamic characteristic estimations revealed that the quadratic model developed using LSM based on three responses is the most accurate among the studied models. Thus, for further investigations of human psychophysiological states based on nonlinear dynamic EMS models derived from three responses, the quadratic IPP model is recommended.

Keywords: information technologies, eye movement system, nonlinear dynamic identification, eye-tracking technology, Volterra models, modeling accuracy, human neurophysiological state diagnostics

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1. INTRODUCTION

Eye-tracking technology has emerged as a powerful tool for evaluating neurocognitive health, particularly in conditions like Alzheimer's [1], [2], [3] and Parkinson's [4] diseases. Studies indicate that eye-tracking data, by detecting characteristic ocular movement patterns, can aid in the early diagnosis of these conditions, offering a non-invasive approach for health monitoring. Eye-tracking has also been applied in posttraumatic stress disorder (PTSD) research, where it helps identify attentional biases toward negative stimuli, contributing to better understanding and potential early diagnosis of the disorder [5]. Furthermore, recent research suggests that eye-tracking can serve as an effective tool for autism diagnosis, with particular biomarkers in eye movements being indicative of autism spectrum disorder [6].

Additionally, eye-tracking technology has shown considerable potential in dyslexia research. By examining specific eye movement patterns in dyslexic individuals, researchers have identified characteristic behaviors that inform targeted interventions. This has paved the way for creating customized educational solutions, aimed at improving reading outcomes and cognitive engagement for dyslexic readers [7], [8].

Eye-tracking is not limited to medical applications; it has gained traction in the fields of education and social sciences. In online learning environments [9], eye movement data can reveal attention patterns and cognitive engagement, allowing for tailored learning interventions that optimize student engagement and minimize cognitive load. Expanding the application of eye-tracking to professional environments has provided insights that improve the efficiency of various work processes [10], [11] and support teamwork in training sessions, especially in complex healthcare environments [12]. Moreover, eye-tracking data holds promise for enhancing security by enabling gaze-based authentication methods for data access control [13], [14].

Research on the application of eye-tracking technology also extends to anomaly detection, where parametric and nonparametric analyses help identify irregular patterns that could indicate cognitive or neurological issues. For instance, detecting anomalies in eye movement can be used in assessments related to mental workload and multitasking capabilities [15], [16]. Such methodologies are not only relevant in healthcare but can also contribute to safety assessments in high-risk industries, supporting the early detection of cognitive strain among workers, particularly in fields requiring high mental acuity [17, 18], [19]. Further developments in eye-tracking have been explored in ocular disease diagnostics through deep learning techniques, where transfer learning models significantly improve classification accuracy for early eye disease detection [20], [21].

This study focuses on simulating the human eye movement system (EMS) using Volterra models, enabling the evaluation of model precision and computational efficiency. By applying nonlinear modeling techniques, we aim to establish a framework that accurately represents human ocular dynamics. This approach allows for determining the most accurate and computationally efficient model, with practical implications for its effectiveness in applications.

2. PROBLEM STATEMENT

To simulate the human eye movement system, integral nonlinear models [22, 23], [24, 25], [26] are employed, which consider both the nonlinear and inertial properties of the system being studied. The EMS is simulated by determining multidimensional transient characteristics (MTCs) based on "input-output" experimental data [25]. Eye-tracking technology is used to collect these experimental data, enabling accurate recording of eye responses to visual stimuli. The construction of the model involves an approximation simulation method using integro-power series (IPS) [26], [27], [28] and a least squares method (LSM) [26], [29, 30], [31] to create the model based on integro-power polynomials (IPP). The simulation methods for nonlinear dynamic systems (NDS) based on IPS and IPP vary in their computational approaches, providing distinct methodologies for NDS simulation [28].

The objective of this research is to rigorously evaluate EMS simulation accuracy, selecting the most precise and computationally efficient nonlinear dynamic model based on IPS and IPP methods. The optimal model will then be used to construct feature spaces tailored to individual psychophysiological profiles, enhancing reliability in assessing personalized conditions. This study also addresses the development of algorithmic and software tools for extracting EMS dynamic characteristics from eye-tracking data and systematically comparing the effectiveness of different simulation methods.

3. THEORETICAL BACKGROUND

In this study, the approximation method [28] and compensation method [26] are employed to develop models using IPS, while the least squares method (LSM) [24] is utilized for constructing models based on IPP.

Approximation Identification Method. The approximation identification method for NDS (method of linear combinations of responses) in the time domain is grounded in isolating the n-th partial component (PC) of the NDS response by constructing linear combinations of responses to test signals with different amplitudes. This approach is an adaptation of methods originally based on the Volterra series. It is proved in [26] that:

Assertion 1. Let test signals $a_1x(t)$, $a_2x(t),\dots,a_Nx(t)$ be sequentially applied to the input of the NDS, where N is the degree; $a_1, a_2,\dots,a_N$ are different real numbers, non-zero, satisfying the condition $|a_j| \le 1$ for $\forall j=1,2,...,N$; $x(t)$ is an arbitrary function. Then, the linear combination of the system's responses to these inputs equals the n-th PC of the response to the input signal $x(t)$ with an accuracy up to the discarded terms Δ of the IPS of order $N+1$ and higher:

$$\sum_{j=1}^{N} c_j y[a_j x(t)] = y_n[x(t)] + \Delta, \tag{1}$$

where

$$y_n[x(t)] = y_n(t),$$

$$y[a_j x(t)] = \sum_{n=1}^{\infty} a_j^n \int_0^t \dots \int_0^t w_n(t-\tau_1,...,t-\tau_n) \prod_{i=1}^{n} x(\tau_i) d\tau_i;$$

$$\Delta = \sum_{j=1}^{N} c_j \sum_{n=N+1}^{\infty} y_n[x(t)],$$

if c_j are real coefficients such that

$$\mathrm{A}_N \, \mathrm{c} = \mathrm{b}, \tag{2}$$

where

$$\mathrm{A}_N = \begin{bmatrix} a_1 & a_2 & \dots & a_N \\ a_1^2 & a_2^2 & \dots & a_N^2 \\ \dots & \dots & \dots & \dots \\ a_1^N & a_2^N & \dots & a_N^N \end{bmatrix}, \quad \mathrm{c} = \begin{bmatrix} c_1 \\ c_2 \\ \dots \\ c_N \end{bmatrix}, \quad \mathrm{b} = \begin{bmatrix} b_1 \\ b_2 \\ \dots \\ b_N \end{bmatrix},$$

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here $b_l = 1$ when $l = n$; $b_l = 0$ when $l \neq n$, $\forall l \in \{1, 2, \ldots, N\}$.

The system (2) always has a solution, and it is unique since its determinant differs from the Vandermonde determinant only by the factor a_1, $a_2, \ldots, a_N$. Thus, for any real numbers a_j, which are non-zero and pairwise distinct, it is possible to find numbers c_j such that the linear combination (1) of the NDS responses equals the n-th term of the IPS with an accuracy up to the discarded terms of the series. By satisfying the conditions for forming the system of linear algebraic equations (2), we obtain the relation (1).

When test signals in the form of step functions (Heaviside functions – $\theta(t)$) with amplitudes a_1, $a_2, \ldots, a_N$ are applied to the input of the system being identified, we obtain estimates of the diagonal cross-sections of the NDS multidimensional transient characteristics:

$$\hat{h}_n(t,...,t) = \hat{y}_n(t) = \sum_{j=1}^{N} c_j^{(n)} y(a_j \theta(t)) = c_1^{(n)} y(t \mid a_1) + \\ + c_2^{(n)} y(t \mid a_2) + \ldots + c_N^{(n)} y(t \mid a_N) + \Delta_n, n = \overline{1, N}; \quad (3)$$

where $y(t \mid a_j) = y(a_j \theta(t))$ are the NDS responses to the test signal with amplitude a_j, and Δ_n represents the methodological error resulting from the discarded terms of the IPS of order $n+1$ and higher.

Fig. 1 shows a structural scheme demonstrating the calculation of the diagonal cross-section of the second-order transient characteristic based on three test signals. This scheme is a particular application of formulas (2) and (3), illustrating how the step test signals are processed through the NDS to yield the transient characteristic.

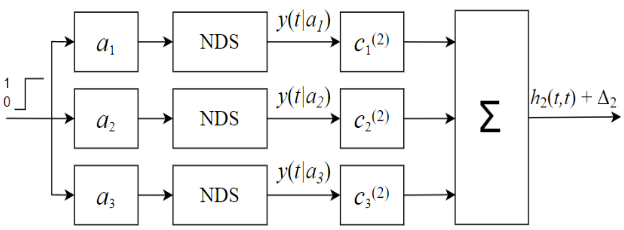

Fig. 1. **Structural scheme for computing the diagonal cross-section of the second-order transient characteristic using the approximation identification method**
Source: **compiled by the authors**

Identification of NDS using the Least Squares Method. The method of NDS identification based on the Volterra polynomial model in the time domain relies on approximating the NDS response $y(t)$ to an arbitrary deterministic signal $x(t)$ in the form of an IPP of N-th order (N – the order of the approximation model):

$$y_N(t) = \sum_{n=1}^{N} \hat{y}_n(t) = \\ = \sum_{n=1}^{N} \underbrace{\int_0^t \ldots \int_0^t}_{n\,times} w_n(t-\tau_1,...,t-\tau_n) \prod_{i=1}^{n} x(\tau_i) d\tau_i. \quad (4)$$

Valid assertion [26].

Assertion 2. Let test signals $a_1x(t)$, $a_2x(t)$, …, $a_Lx(t)$ be sequentially applied to the input of the NDS; a_1, $a_2, \ldots, a_L$ are different real numbers satisfying the condition $0 < |a_j| \leq 1$ for $\forall j = 1,2,...,L$; $x(t)$ is an arbitrary deterministic signal, then

$$\tilde{y}_N(a_j x(t)) = \sum_{n=1}^{N} \hat{y}_n(a_j x(t)) = \\ = \sum_{n=1}^{N} a_j^n \underbrace{\int_0^t \ldots \int_0^t}_{n\,times} w_n(t-\tau_1,...,t-\tau_n) \prod_{i=1}^{n} x(\tau_i) d\tau_i = \\ = \sum_{n=1}^{N} a_j^n \hat{y}_n(t) \text{ for } \forall j,\ j = \overline{1, L}, L \geq N. \quad (5)$$

The partial components in the approximation model $\hat{y}_n(t)$ are found using the least squares method. This allows obtaining estimates for them, where the sum of squares of deviations of the NDS responses being identified, $y[a_j x(t)]$ from the model responses $\tilde{y}_N[a_j x(t)]$, is minimal, thus ensuring the minimum mean square criterion

$$J_N = \sum_{j=1}^{L} \left(y(a_j x(t)) - \tilde{y}_N(a_j x(t)) \right)^2 = \\ = \sum_{j=1}^{L} \left(y(t \mid a_j) - \sum_{n=1}^{N} a_j^n \hat{y}_n(t) \right)^2 \to \min. \quad (6)$$

Minimizing criterion (6) boils down to solving a system of normal equations Gauss, which in vector-matrix form can be expressed as:

$$\mathrm{A}'\mathrm{A}\hat{\mathrm{y}} = \mathrm{A}'\,\mathrm{y}, \quad (7)$$

where

$$\mathrm{A} = \begin{bmatrix} a_1 & a_1^2 & \cdots & a_1^N \\ a_2 & a_2^2 & \cdots & a_2^N \\ \cdots & \cdots & \cdots & \cdots \\ a_L & a_L^2 & \cdots & a_L^N \end{bmatrix}, \quad \mathrm{y} = \begin{bmatrix} y(t \mid a_1) \\ y(t \mid a_2) \\ \cdots \\ y(t \mid a_L) \end{bmatrix}, \quad \hat{\mathrm{y}} = \begin{bmatrix} \hat{y}_1(t) \\ \hat{y}_2(t) \\ \cdots \\ \hat{y}_N(t) \end{bmatrix}.$$

If the identified system is supplied with test signals in the form of step functions with amplitudes a_1, $a_2, \ldots, a_L$, we obtain estimates of the transient characteristics $\hat{h}_1^{(N)}(t)$ and the diagonal cross-sections of the transient characteristics of the eye movement system $\hat{h}_2^{(N)}(t,t)$, $\hat{h}_3^{(N)}(t,t,t)$,…, $\hat{h}_N^{(N)}(t,...,t)$ [26], [28].

Responses of the investigated EMS models are generally calculated based on expressions:

$$\tilde{y}_j(t\,|\,a_j) = a_j\hat{y}_1(t) + a_j^2\hat{y}_2(t) + ... + a_j^N\hat{y}_N(t), \quad j=\overline{1,L}, \quad (8)$$

or

$$\tilde{y}(t\,|\,a_j) = a_j\hat{h}_1^{(N)}(t) + a_j^2\hat{h}_2^{(N)}(t,t) + + ... + a_j^N\hat{h}_N^{(N)}(t,...,t), \quad j=\overline{1,L} \quad . \quad (9)$$

Fig. 2 presents the structural scheme, as described in [26], for calculating the transient characteristics based on the LSM. The scheme represents a second-order model, where block T_2 computes the first- and second-order transient characteristics based on the system of normal equations (7). The matrix A_2 in block T_2 incorporates all three input step signals (a_1, a_2, a_3) in the form of Heaviside function.

The operation of block T_2 and the matrix A_2 are described by the following formulas:

$$T_2 = \left(A_2' A_2\right)^{-1} A_2' \,;\; A_2 = \begin{bmatrix} a_1 & a_1^2 \\ a_2 & a_2^2 \\ a_3 & a_3^2 \end{bmatrix}.$$

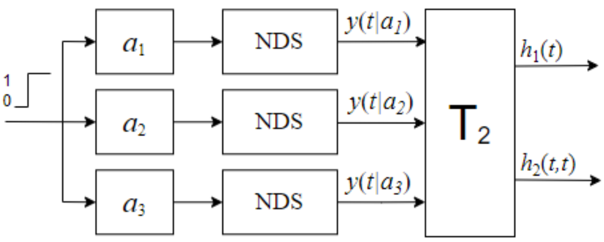

Fig. 2. **Structural scheme for computing the first- and second-order transient characteristics using the LSM identification method**
Source: **compiled by the authors**

Compensation Identification Method. The formalism of the method for determining the intersections of n-th order transient characteristics of nonlinear dynamic systems is based on the following assertion [26], [28].

Assertion 3. Let the test inputs be the sum of n step signals $x_k(t) = a_k\theta(t-\tau_k)$ (k=1, 2,...,n), shifted in time by τ_1, ..., τ_n. Then, for an NDS with a single input and a single output, the estimate of the intersection of the n-th order transient characteristic is

$$\hat{h}_n(t-\tau_1,...,t-\tau_n) = = \left(n!\prod_{k=1}^{n} a_k\right)^{-1} \sum_{\delta_1,...,\delta_n=0}^{1} (-1)^{n+\sum_{k=1}^{n}\delta_k}\, y(t\,|\,\delta_1,...,\delta_n) \quad (10)$$

where $y(t\,|\,\delta_1,...,\delta_n)$ is the NDS response at time t when subjected to a multi-step input signal with amplitudes a_k, obtained as a result of processing experimental data based on (10). If $\delta_k = 1$, the test input contains a step signal shifted by τ_k; otherwise, if $\delta_k = 0$, it does not contain it.

In certain cases, we have:

for n=1

$$\hat{h}_1(t) = \frac{y(t\,|a_1)}{a_1} + \Delta_1; \quad (11)$$

for n=2

$$\hat{h}_2(t,t) = \frac{1}{2a_1^2}\left[y(t\,|a_2) - 2y(t\,|\,a_1)\right] + \Delta_2; \quad (12)$$

or

$$\hat{h}_2(t,t) = \frac{1}{2a_1a_2}\left[y(t\,|a_3) - y(t\,|\,a_1) - y(t\,|a_2)\right] + \Delta_2; \quad (13)$$

for n=3

$$\hat{h}_3(t,t,t) = \frac{1}{6a_1^3}\left[y(t\,|a_3) - 3y(t\,|\,a_2) + 3y(t\,|\,a_1)\right] + \Delta_3. \quad (14)$$

where a_1, $a_2 = 2a_1$, $a_3 = a_1 + a_2$ are the amplitudes of test signals; Δ_n represents the methodological error resulting from the discarded terms of the IPS of order n+1 and higher.

The responses of the second and third-order models are calculated accordingly using the expressions:

$$\tilde{y}(t\,|\,a_j) = a_j\hat{h}_1(t) + a_j^2\hat{h}_2(t,t)\,, \quad (15)$$

$$\tilde{y}(t\,|\,a_j) = a_j\hat{h}_1(t) + a_j^2\hat{h}_2(t,t) + a_j^3\hat{h}_3(t,t,t)\,, \forall j = \overrightarrow{1,L}\,. \quad (16)$$

The compensation identification method is presented with a structural scheme illustrating the calculations based on formula (12), as shown in Fig. 3. This scheme demonstrates the process of obtaining of the diagonal cross-section of the second-order transient characteristics using two test signals with amplitudes a_1 and a_2 in the form of the Heaviside function. In Fig. 4, the same scheme is shown for the transient characteristic obtained using three test signals in the form of the Heaviside function, with amplitudes a_1, a_2 and a_3, as described in formula (13).

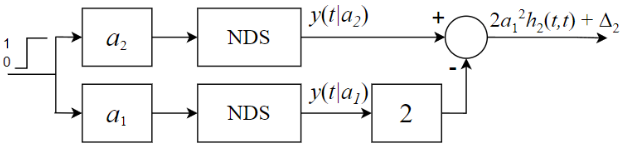

Fig. 3. **Structural scheme for computing the diagonal cross-section of the second-order transient characteristic using the compensation method with two test signals**
Source: **compiled by the authors**

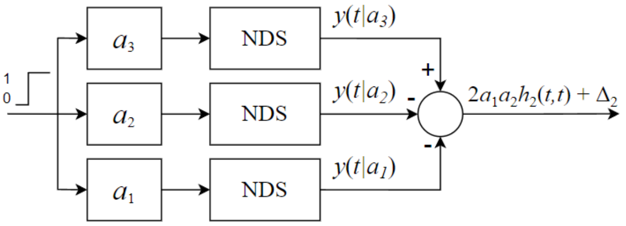

Fig. 4. **Structural scheme for computing the diagonal cross-section of the second-order transient characteristic using the compensation method with three test signals**
Source: **compiled by the authors**

4. ACCURACY ANALYSIS OF EYE MOVEMENT SYSTEM SIMULATION MODELS

The EMS's responses to the test step signals, defined as $x(t) = a_j\theta(t)$ with amplitudes a_j (j=1, 2, 3): a_1=1/3, a_2=2/3, a_3=1 were analyzed. These responses formed the basis in the construction of Volterra models [24]. Horizontal visual stimuli displayed at varying distances from the starting position on a monitor were utilized as test signals, effectively simulating the application of step signals with different amplitudes to the EMS. The responses of the EMS were recorded using eye-tracking technology, integrating both hardware and software components. In the simulation process, when applying the approximation method, models based on IPS are identified as M1.N/x:<$a_1,\ldots, a_L$> (N – order of approximation, x – number of test signals; $a_1,\ldots, a_L$ – amplitudes of the test step signals). For models based on IPP, the least squares method (LSM) is used, resulting in models designated as M2.N/x:<$a_1,\ldots, a_L$>. Additionally, when employing the compensation method of simulation, models are determined as M3.N/x:<$a_1,\ldots, a_L$>.

For EMS simulation in works [28]–[31], experimental "input-output" data were gathered using three test step signals with amplitudes a_1=1/3, a_2=2/3 and a_3=1. The Tobii Pro TX300 eye tracker was employed to collect these experimental data (Fig. 5), from which transient characteristics were determined for models M1.N, M2.N, and M3.N for N=1 (linear model), N=2 (quadratic model), and N=3 (cubic model). The transient processes of EMS responses to visual stimuli with varying amplitudes are depicted in Fig. 6.

The software tools were developed using the Python programming environment.

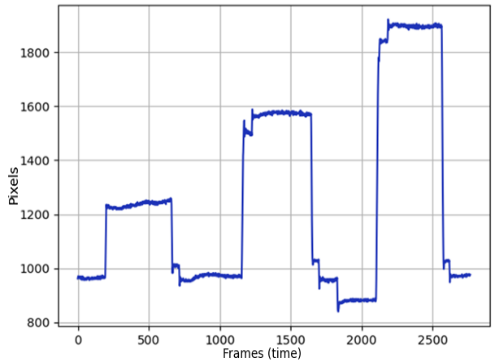

Fig. 5. **EMS responses to visual stimuli of different amplitudes**
Source: **compiled by the authors**

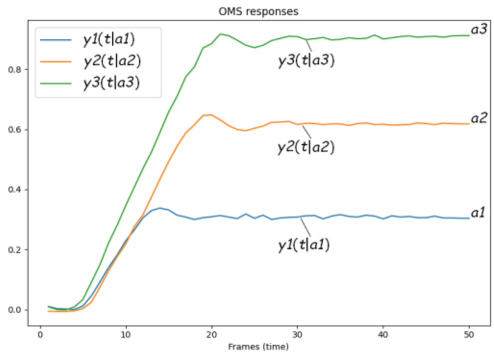

Fig. 6. **Transient processes of EMS responses to visual stimuli of different amplitudes**
Source: **compiled by the authors**

To evaluate the accuracy of the developed models for varying amplitudes of the test signals a_1, a_2 and a_3, the metric applied is the normalized root mean square error (NRMSE):

$$\varepsilon_{a_j} = \left(\frac{\sum_{m=0}^{M} \left(y(t_m \mid a_j) - \tilde{y}(t_m \mid a_j) \right)^2}{\sum_{m=0}^{M} y^2(t_m \mid a_j)} \right)^{1/2}, \quad j=1, 2, 3; \qquad (17)$$

where $y(t_m \mid a_j)$ and $\tilde{y}(t_m \mid a_j)$ are the responses of the EMS and the model of the EMS to the test signal in the form of a step function with amplitude a_j, measured/ computed at the time instant t_m (t_m is the observation time of the EMS responses); j=1,2,3.

For N=1, transient characteristics $\hat{h}_1(t \mid a_j)$ (j=1, 2, 3) were obtained based on the responses $y(t \mid a_1)$ or $y(t \mid a_2)$ or $y(t \mid a_3)$, as shown in Fig. 7.

Fig. 8 shows transient characteristics graphs of the M2.1/2 models, which were calculated based on two responses: $y(t\,|\,a_1)$ and $y(t\,|\,a_2)$, or $y(t\,|\,a_1)$ and $y(t\,|\,a_3)$, or $y(t\,|\,a_2)$ and $y(t\,|\,a_3)$ and the transient characteristic graph of the model M2.1/3. For the M1.1 identification method does not allow to calculate the transient characteristics based on two or three responses [26].

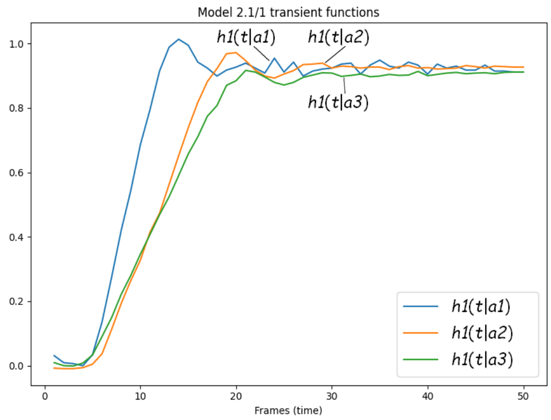

Fig. 7. **Transient characteristics of the EMS models M1.1/1 and M2.1/1, built using test signals with amplitudes *a*₁; *a*₂; *a*₃**
Source: **compiled by the authors**

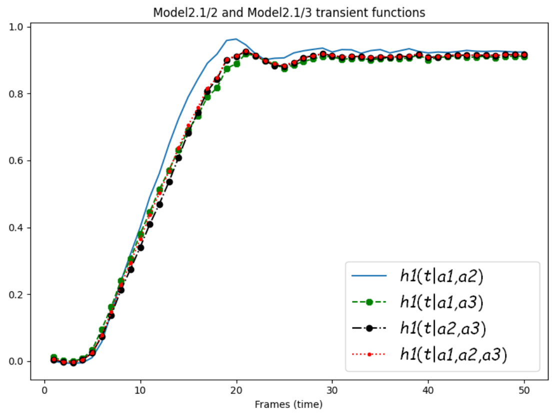

Fig. 8. **Transient characteristics of the EMS models M2.1/2, built using test signals with amplitudes: a_1 and a_2; a_1 and a_3; a_2 and a_3; and M2.1/3**
Source: **compiled by the authors**

For the compensation identification method, the model M3.1 based on the response $y(t\,|\,a_1)$ was derived. The corresponding transient characteristic is illustrated in Fig. 9, while the model's responses are depicted in Fig. 10.

Table 1 shows the accuracy values based on percentage NRMSE estimates of the responses obtained using the identification methods for EMS models M1.1/1, M2.1/1, and M3.1, while Table 2 presents the values for models M2.1/2 and M2.1/3.

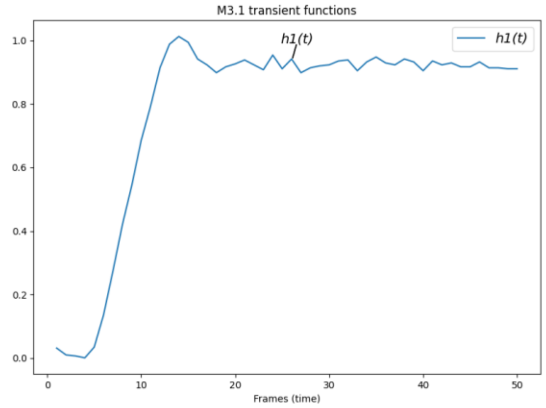

Fig. 9: **Transient characteristic of the EMS model M3.1 built using test signal with amplitude a_1**
Source: **compiled by the authors**

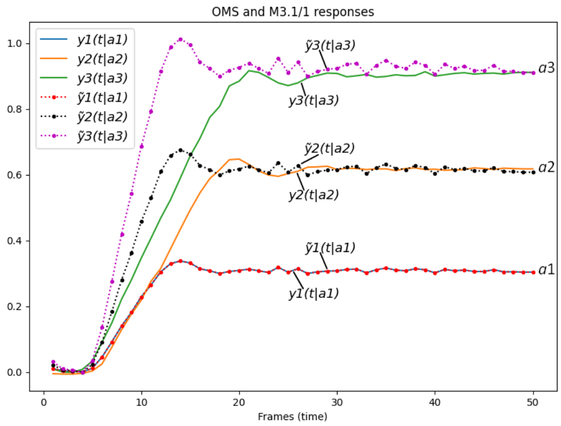

Fig. 10. **Responses of the EMS and the model M3.1 built using test signal with amplitude a_1**
Source: **compiled by the authors**

Table 1. **Accuracy based on Percentage Normalized Root Mean Square Error of the EMS models M1.1/1, M2.1/1 and M3.1 %**

Models	Amplitudes of test signals			Mean value
	a_1	a_2	a_3	
M1.1/1:a_1	99.9	81.8	79.7	80.8
M1.1/1:a_2	82.8	99.9	94.5	88.6
M1.1/1:a_3	81.5	94.7	99.9	88.1
M3.1:a_1	99.9	82	80	81

Source: **compiled by the authors**

Table 2. **Accuracy Based on Percentage Normalized Root Mean Square Error of the EMS models M2.1/2 and M2.1/3 %**

Models	Amplitudes of test signals			Mean value
	a_1	a_2	a_3	
M2.1/2:a_1,a_2	86.2	96.4	92.9	91.8
M2.1/2:a_1,a_3	83.3	95.1	98.0	92.1
M2.1/2:a_2,a_3	82.0	96.3	98.3	92.2
M2.1/3:a_1,a_2,a_3	83.3	96.5	97.5	92.4

Source: **compiled by the authors**

For N=2, based on the two responses $y(t \mid a_1)$ and $y(t \mid a_2)$, or $y(t \mid a_1)$ and $y(t \mid a_3)$, or $y(t \mid a_2)$ and $y(t \mid a_3)$, the corresponding multidimensional transient characteristics $\hat{h}_1(t|a_j,a_k)$ and $\hat{h}_2(t,t|a_j,a_k)$, $j, k = 1,2,3; j \neq k$ were obtained. In the models M1.2/2 and M2.2/2, identical MTCs $\hat{h}_1(t|\, a_j,a_k)$ and $\hat{h}_2(t\,,t|\, a_j,a_k)$ were obtained for the same experimental data.

Fig. 11 displays both the first-order transient characteristics and the diagonal cross-sections of the second-order MTCs for the EMS models M1.2/2 and M2.2/2. Both the first- and second-order transient characteristics were computed based on the responses: $y(t \mid a_1)$ and $y(t \mid a_2)$, or $y(t \mid a_1)$ and $y(t \mid a_3)$, or $y(t \mid a_2)$ and $y(t \mid a_3)$.

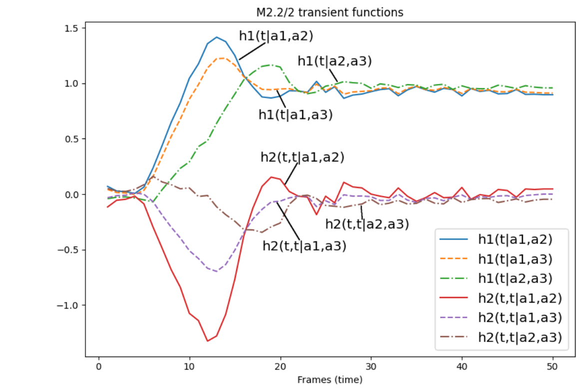

Fig. 11. **Transient characteristics of the first-order and diagonal cross-sections of the second-order of the EMS, models M1.2/2 and M2.2/2**
Source: **compiled by the authors**

The transient characteristics of the EMS model M3.2/2 are illustrated in Fig. 12. The corresponding responses for the same model are given in Fig. 13.

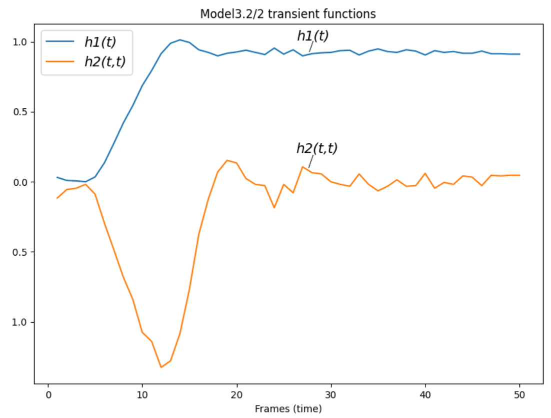

Fig. 12. **Multidimensional transient characteristics of the EMS, model M3.2/2**
Source: **compiled by the authors**

The graphs of the multidimensional transient characteristics $\hat{h}_1(t)$ and $\hat{h}_2(t,t)$, which were determined based on the three responses $y(t \mid a_1)$, $y(t \mid a_2)$, $y(t \mid a_3)$ for the EMS model M2.2/3, are shown in Fig. 14.

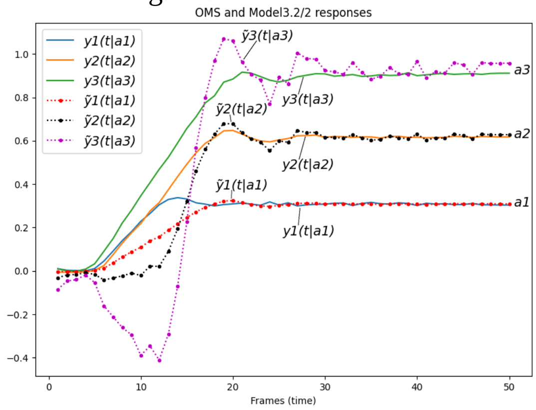

Fig. 13. **Responses of the EMS and the model M3.2/2**
Source: **compiled by the authors**

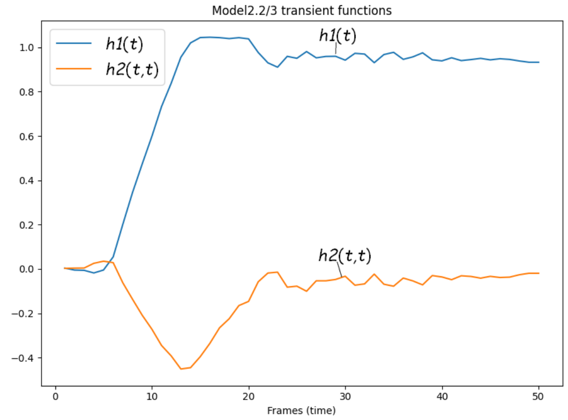

Fig. 14. **Multidimensional transient characteristics of the EMS, model M2.2/3**
Source: **compiled by the authors**

The corresponding graphs comparing the M2.2/3 responses with the EMS responses to identical test signals are presented in Fig. 15. Analogous results were obtained for the M3.2/3 and are shown in Fig. 16 and Fig. 17.

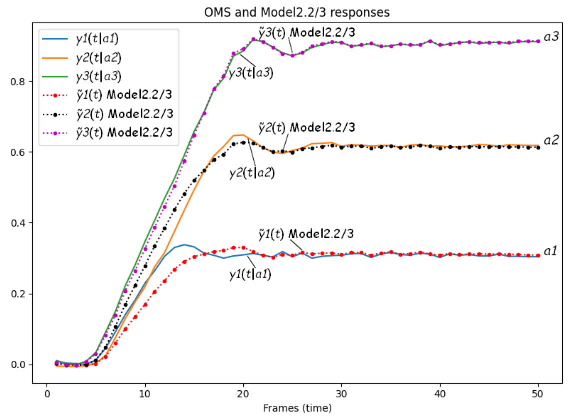

Fig. 15. **Responses of the EMS and the model M2.2/3**
Source: **compiled by the authors**

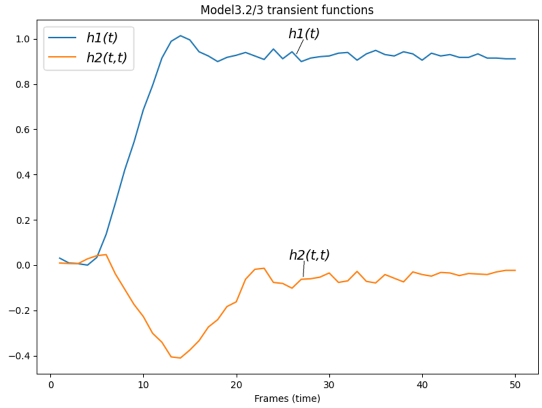

Fig. 16. **Multidimensional transient characteristics of the EMS, model M3.2/3**

Source: **compiled by the authors**

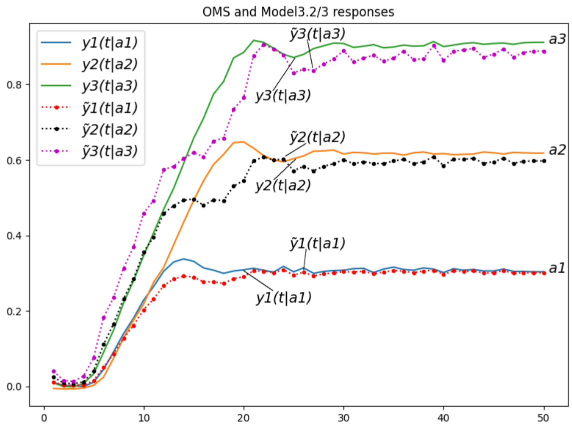

Fig. 17. **Responses of the EMS and the model M3.2/3**

Source: **compiled by the authors**

For N=3, the graph of the primary-order transient characteristic, along with the graphs of the diagonal cross-sections of the second and third-order multidimensional transient characteristics for the EMS model M3.3, are displayed in Fig.18. The responses of the EMS model M3.3 are shown in Fig.19.

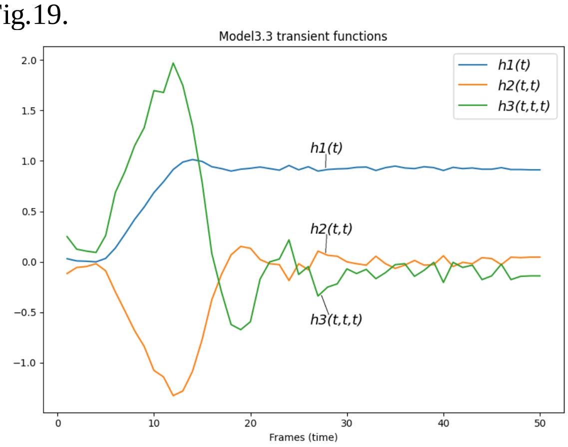

Fig. 18. **Multidimensional transient characteristics of the EMS model M3.3**

Source: **compiled by the authors**

Table 3 provides the accuracy values based on percentage normalized root mean square error of the response estimates for the constructed EMS models M1.2/2 and M2.2/2, and the model M2.2/3. Table 4 presents the accuracy values for the models M3.2/2 and M3.2/3, and the model M3.3/3.

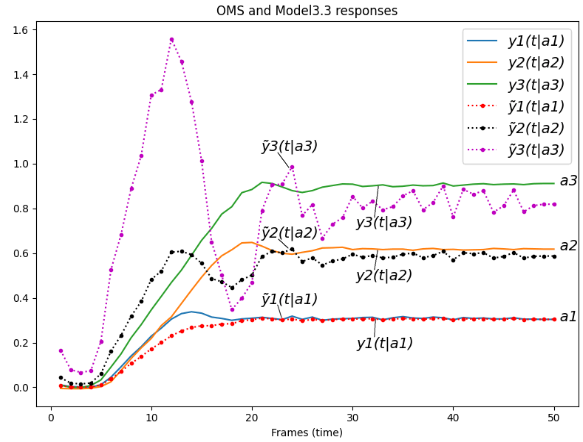

Fig. 19. **Responses of the EMS and the model M3.3**

Source: **compiled by the authors**

Table 3. **Accuracy Based on Percentage Normalized Root Mean Square Error of the EMS models M1.2/2, M2.2/2 and M2.2/3, %**

Model	Amplitudes of test signals			Mean value
	a_1	a_2	a_3	
M2.2/2:a_1,a_2	99.9	99.9	81.0	81.0
M2.2/2:a_1,a_3	99.9	90.9	99.9	90.9
M2.2/2:a_2,a_3	82.7	99.9	99.9	82.7
M2.2/3:a_1,a_2,a_3	91.8	95.7	99.0	95.5

Source: **compiled by the authors**

Table 4. **Accuracy Based on Percentage Normalized Root Mean Square Error of the EMS models M3.2/2, M3.2/3 and M3.3, %**

Model	Amplitudes of test signals			Mean value
	a_1	a_2	a_3	
M3.2/2:a_1,a_2	82.8	81.8	62.6	75.7
M3.2/3:a_1,a_2,a_3	93.9	88.7	92.2	91.6
M3.3/3:a_1,a_2,a_3	90.7	80.1	51.4	74.1

Source: **compiled by the authors**

For N=3, based on the three responses $y(t \mid a_1)$, $y(t \mid a_2)$, $y(t \mid a_3)$ the transient characteristics $\hat{h}_1(t)$, $\hat{h}_2(t,t)$, $\hat{h}_3(t,t,t)$ were obtained. For the models M1.3/3 and M2.3/3, identical MTCs were obtained, and the responses of the models practically coincide with the responses of the EMS for the same input signals.

On Fig. 20, a comparative analysis diagram of the accuracy based on the percentage NRMSE criterion constructed using identification software tools for EMS models: M2.1/1, M2.1/2, M2.1/3, is presented.

On Fig. 21, the same analysis is provided for models M2.2/2, M3.2/2, M2.2/3, M3.2/3 and M3.3. The EMS model M2.3 are not shown in the diagram because it have negligible deviations from the EMS responses.

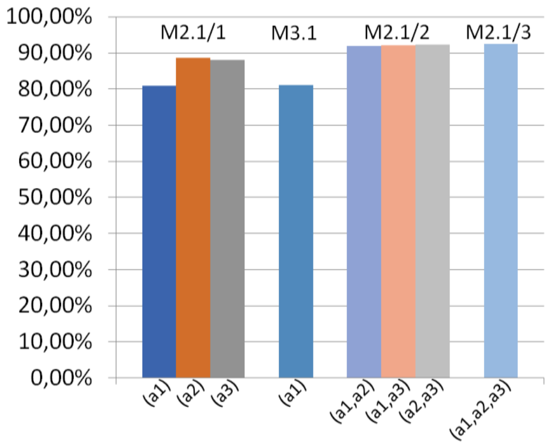

Fig. 20. **Comparative analysis of the accuracy based on the average percentage NRMSE values of the EMS models:**
M2.1/1; M3.1; M2.1/2; M2.1/3
***Source:* compiled by the authors**

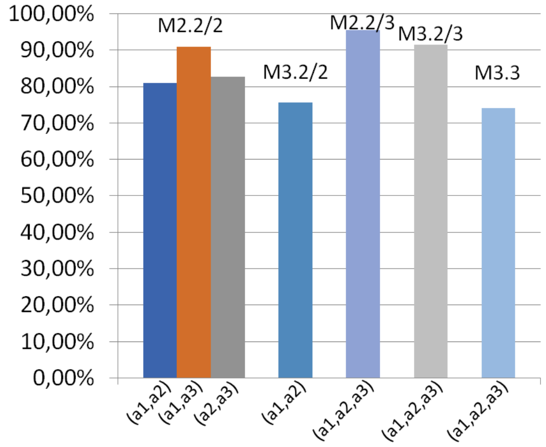

Fig. 21. **Comparative analysis of the accuracy based on the average percentage NRMSE values of the EMS models:**
M2.2/2; M3.2/2; M2.2/3; M3.2/3; M3.3
***Source:* compiled by the authors**

5. CONCLUSION

This study applied Python-implemented nonlinear dynamic simulation of the human EMS, utilizing integro-power series and integro-power polynomials to develop models that accurately reflect the eye movement system's response dynamics. By employing the least squares method, compensation, and approximation methods, we evaluated various model accuracies based on first-order transient characteristics and diagonal cross-sections of second- and third-order multidimensional transient characteristics. The data for these models was collected from eye-tracking experiments designed to capture responses to test signals with varying amplitudes.

The results demonstrate that the quadratic model, developed using three test signal responses and refined with the least squares method, provided the highest simulation accuracy and computational efficiency, outperforming alternative models in precision. The evaluation showed a significant reduction in error rates as more test signals were incorporated, highlighting the value of using multiple data points for improved accuracy. While the compensation method required fewer computational resources, it exhibited higher error rates, making it less suitable for applications requiring high diagnostic precision. It is important to note that the models of the third order and the second-order model based on the compensation method using two test signals yield unstable solutions.

In summary, the quadratic model using the least squares method and three test signals provides a reliable framework for EMS-based state classification. The developed models, particularly the quadratic IPP model, offer a robust foundation for further research into personalized psychophysiological condition assessment through the development of classifiers. This methodology enables high accuracy in EMS simulation and effective psychophysiological condition assessment, providing a strong basis for future studies and EMS-related applications.

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Оцінки точності ідентифікації око-рухової системи людини за допомогою ступінчатих тестових сигналів

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АНОТАЦІЯ

Для математичного моделювання око-рухової системи (ОРС) людини використовуються інтегральні нелінійні моделі, які одночасно враховують нелінійну динаміку та інерційні властивості об'єкта дослідження. На основі даних експериментальних досліджень ОРС "вхід-вихід" визначаються діагональні перетини багатовимірних перехідних характеристик (БПХ) другого та третього порядків. Для отримання експериментальних даних застосовується інноваційна технологія айтрекінгу, що дозволяє реєструвати відгуки ока на тестові візуальні стимули. Мета роботи полягає в дослідженні точності ідентифікації ОРС за даними айтрекінгу шляхом оцінки похибок обчислення БПХ при використанні трьох методів нелінійної динамічної ідентифікації: апроксимаційного, компенсаційного та методу найменших квадратів (МНК) на основі моделей у вигляді інтегро-степеневих рядів (ІСР) та інтегро-степеневих поліномів (ІСП). Об'єктом дослідження є процес непараметричної ідентифікації ОРС на основі моделей Вольтерри у часовій області. Предметом дослідження є обчислювальні та програмні засоби визначення динамічних характеристик ОРС за даними айтрекінгу, аналіз точності отриманих моделей при використанні зазначених методів. Отримано оцінки точності побудованих різних моделей ОРС (лінійної, квадратичної та кубічної) за даними трьох відгуків на тестові сигнали різної амплітуди. Для апроксимаційного методу та МНК при використанні однакових тестових сигналів було отримано однакові моделі, оскільки в області збіжності ІСР ці моделі співпадають. Компенсаційний метод вимагає мінімальних обчислювальних ресурсів порівняно з іншими методами, проте моделі, побудовані при його використанні, мають значні похибки, що робить їх непридатними в діагностичних дослідженнях.При побудові моделей третього порядку проявляється нестабільність отриманих оцінок перехідних характеристик. На основі аналізу похибок оцінки динамічних характеристик ОРС встановлено, що квадратична модель побудована за допомогою МНК на основі трьох відгуків є найкращою серед досліджуваних моделей. Таким чином, у подальших дослідженнях психофізіологічного стану людини на основі нелінійних динамічних моделей ОРС за даними трьох відгуків доцільно використовувати модель у вигляді квадратичного ІСП.

Ключові слова: інформаційні технології, око-рухова система, нелінійна динамічна ідентифікація, технологія айтрекінгу, моделі Вольтерри, точність моделювання, діагностика нейрофізіологічних станів людини

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