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Face recognition using ten-variate prediction ellipsoids for normalized data with different quantiles

Sergiy B. Prykhodko[1),2)]
ORCID: https://orcid.org/0000-0002-2325-018X; sergiy.prykhodko@nuos.edu.ua. Scopus Author ID: 55225622100
Artem S. Trukhov[1)]
ORCID: https://orcid.org/0000-0002-7160-8609; artem.trukhov@gmail.com
[1)] Admiral Makarov National University of Shipbuilding, 9, Heroes of Ukraine Ave. Mykolaiv, 54007, Ukraine
[2)] Odessa Polytechnic National University, 1, Shevchenko Ave. Odessa, 65044, Ukraine

ABSTRACT

Facial recognition technology plays a pivotal role in various domains, including security systems, entertainment, and identity verification. However, the low probability of identifying a person by face can have negative consequences, highlighting the need for the development and improvement of face recognition methods. The object of research is the face recognition process, with the subject of the research being a mathematical model for face recognition. One common approach in pattern recognition is using decision rules based on prediction ellipsoid. A significant challenge in its application is ensuring that the data conforms to a multivariate normal distribution. However, real-world data often doesn't adhere to this assumption, leading to reduced recognition probability. Therefore, there's a necessity to enhance mathematical models to accommodate such deviations. Another factor that can impact the outcome is the selection of different distribution quantiles, such as those from the Chi-square and F-distribution. For large datasets, the utilization of Chi-square and F-distribution in prediction ellipsoids typically results in similar probabilities, but there are data for which this is not the case and the application of prediction ellipsoids with different quantiles of the distributions gives different results. This study investigates the application of prediction ellipsoids in facial recognition tasks using different normalization techniques and distribution quantiles. The purpose of the work is to improve the probability of face recognition by building a ten-variate prediction ellipsoid for normalized data with different quantiles of distributions. We conducted experiments on a dataset of facial images and constructed prediction ellipsoids based on the Chi-square and F-distribution, utilizing both univariate and multivariate normalization techniques. Our findings reveal that normalization techniques significantly enhance recognition accuracy, with multivariate methods, such as the ten-variate Box-Cox transformation, outperforming univariate approaches. Furthermore, prediction ellipsoids constructed using the Chi-square distribution quantile generally exhibit superior performance compared to those constructed using the F-distribution quantile. Future investigations could explore the efficacy of alternative normalization techniques, such as the Johnson transformation, and analyze the construction of prediction ellipsoids with alternative components of the ellipsoid equation.
Keywords: Facial recognition; multivariate normal distribution; Chi-square; F-distribution; prediction ellipsoid; data normalization; Box-Cox transformation

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INTRODUCTION

Facial recognition is a rapidly growing technology that has many applications in various domains such as computer vision, security systems, and others. It is a technique that automatically recognizes individuals by analyzing their unique facial features, such as the shape of the eyes, nose, mouth, and other attributes. Facial recognition technology is constantly evolving and improving, resulting in higher accuracy and enabling new possibilities for its use in different aspects of life [1].

The performance and reliability of facial recognition systems depend largely on the specific decision rule that is used for the recognition process. This decision rule determines how the facial features of an individual are assigned to predefined categories or classes within the system.

One of the main challenges in current methods of face recognition is the assumption that the data follows a multivariate normal distribution. However, this is often not the case, as real data can have a non-Gaussian distribution. This can lead to errors in the face recognition process. Therefore, there is a need to develop mathematical models that can account for deviations from the normal distribution of data.

The object of study is the face recognition process, which consists of several key steps: image preprocessing, feature extraction, and pattern recognition [2], which uses mathematical models to determine which individual the feature vector matches.

The subject of study is a mathematical model for face recognition. One of the common methods in pattern recognition is to construct decision rules based on prediction ellipsoids.

The purpose of the work is to improve the probability of face recognition by building a ten-variate prediction ellipsoid for normalized data with different quantiles of distributions.

ANALYSIS OF LITERARY DATA

Traditionally, face recognition systems have commonly employed classification techniques, aiming to assign input face images to predefined categories, such as linear discriminate analysis [3, 4], support vector machine [5], and principal component analysis combined with face recognition by using K-Nearest-Neighbor [6] and neural networks [7], including convolutional neural networks [8].

However, these methods are ineffective in situations where there are only representatives of one class. To overcome this limitation, the one-class classification approach has emerged [9]. This method, similar to anomaly detection, focuses on learning representations of a single class, usually the target one, without explicit knowledge of other classes, treating representatives of these other classes as anomalies.

Within the realm of one-class classification, both prediction ellipsoid [10] and neural network-based methods [11, 12] stand out as popular choices. Prediction ellipsoid methods offer simplicity in implementation and provide interpretability that facilitate result interpretation. In contrast, neural network-based approaches, while powerful, often lack transparency in their decision-making process, making it challenging to discern the factors influencing classification outcomes.

A prediction ellipsoid is a region in multivariate space within which future observations are expected to fall with a certain level of confidence. The center of the ellipsoid is the mean vector, and the size and orientation of the ellipsoid are determined by the covariance matrix and the confidence level [13].

Face recognition is facilitated by the prediction ellipsoid, which delineates the allowable space for each class, containing elements within the same class while excluding those from other classes. The left part of the equation represents the squared Mahalanobis distance (SMD), a measure of how far a point is from the mean, considering the covariance structure of the data. Constructing a prediction ellipsoid involves calculating SMD for each observation, and determining critical values from the Chi-square or F-distribution based on the desired confidence level and number of dimensions [14].

Therefore, the probability that a point lies inside the ellipsoid is equal to the probability that the SMD is less than or equal to a constant. This constant can be derived from the quantile of the Chi-square or F-distribution for a given confidence level. The value of the SMD follows a Chi-squared distribution with k degrees of freedom, where k is the number of characteristics [15]. In this case, the prediction ellipsoid has a form:

$$\left(\mathrm{X}-\overline{\mathrm{X}}\right)^T \mathrm{S}_{\mathrm{X}}^{-1}\left(\mathrm{X}-\overline{\mathrm{X}}\right)=\chi^2_{k,\alpha}, \qquad (1)$$

where

$$\mathrm{S}_{\mathrm{X}}=\frac{1}{N}\sum_{i=1}^{N}\left(\mathrm{X}_i-\overline{\mathrm{X}}\right)\left(\mathrm{X}_i-\overline{\mathrm{X}}\right)^T .$$

Prediction ellipsoids can also be constructed using the squared Mahalanobis distance and the F-distribution. The F-distribution is a ratio of two independent Chi-square distributions, divided by their degrees of freedom. The value of the constant is then given by a formula that involves the F-distribution [16]:

$$\left(\mathrm{X}-\overline{\mathrm{X}}\right)^T \mathrm{S}_{\mathrm{X}}^{-1}\left(\mathrm{X}-\overline{\mathrm{X}}\right)=\frac{k\left(N^2-1\right)}{N\left(N-k\right)}F_{k,N-k,\alpha} . \qquad (2)$$

The creation of the ellipsoid is based on the presumption that the data conforms to a multivariate normal distribution [17]. The Mardia test [18] is used to check whether the data deviate from normality. Among the data that may deviate from Gaussian distribution are the facial metric samples examined in this study. To address practical challenges associated with the Mahalanobis distance when dealing with non-Gaussian data, normalization techniques are employed. These techniques facilitate problem resolution for data with multivariate distributions departing from normality [19].

Among the most widely used data normalization techniques are categorized into two main types: univariate and multivariate transformations. Univariate transformations, while simpler to apply, overlook the correlations between features, often resulting in inferior outcomes. Examples include the univariate logarithmic transformation and the univariate Box-Cox transformation (BCT) [20]. On the other hand, the multivariate Box-Cox transformation, although more complex to implement, tends to yield superior results. A distinctive aspect of this study is the novel application of multidimensional normalization transformation in the realm of face recognition.

When dealing with large datasets, both the Chi-square and F-distribution tend to approach the normal distribution due to the central limit theorem. As a result, their behavior becomes increasingly similar, leading to comparable probabilities when constructing prediction ellipsoids. This phenomenon occurs because the Chi-square distribution is derived from the sum of squared standard normal variables, while the F-distribution arises from the ratio of two independent Chi-square variables [21]. In practice, researchers often observe converging results between the two distributions when analyzing large datasets, making them interchangeable in many cases for constructing prediction ellipsoids

Nevertheless, in practice, there are cases when the application of different quantiles of the distributions for construction prediction ellipsoids gives different results, so it makes sense to check this and compare the results of the application of the prediction ellipsoids for normalized data based on Chi-square and F-distributions.

FORMAL PROBLEM STATEMENT

Suppose given the original data sample set of the ten geometrical facial features the multivariate distribution for which is not Gaussian. Suppose that there are bijective ten-variate normalizing transformation $\psi = \{\psi_Y, \psi_1, \psi_2, \ldots, \psi_{10}\}^T$ of non-Gaussian random vector $X = \{X_1, X_2, \ldots, X_{10}\}^T$ to Gaussian random vector $Z = \{Z_1, Z_2, \ldots, Z_{10}\}^T$ is given by:

$$Z = \psi(X), \quad (3)$$

and the inverse transformation for (1)

$$X = \psi^{-1}(Z). \quad (4)$$

It is required to build the prediction ellipsoid for normalized data using the Chi-squared distribution, based on (1):

$$\left(Z - \overline{Z}\right)^T S_Z^{-1}\left(Z - \overline{Z}\right) = \chi^2_{m,\alpha}, \quad (5)$$

and using F-distribution, based on (2):

$$\left(Z - \overline{Z}\right)^T S_Z^{-1}\left(Z - \overline{Z}\right) = \frac{k\left(N^2 - 1\right)}{N\left(N - k\right)} F_{k,N-k,\alpha}, \quad (6)$$

where:

$$S_Z = \frac{1}{N}\sum_{i=1}^{N}\left(Z_i - \overline{Z}\right)\left(Z_i - \overline{Z}\right)^T.$$

Also, it is required to develop the decision rules for face recognition based on equations (5), (6), and the transformations (3) and (4) and compare the results.

MATERIALS AND RESEARCH METHODS

To generate feature vectors, a custom Python script was developed utilizing the Dlib computer vision framework [22]. Upon identifying a face within the source image, the script executes a series of image manipulation procedures, including facial cropping and alignment to standardize eye-level positioning. These preprocessing techniques aim to mitigate distortions arising from variations in facial orientation [23]. Finally, the script extracts a collection of attributes from the aligned facial image, with each attribute representing the pixel-based distance between predefined facial landmarks provided by the Dlib library.

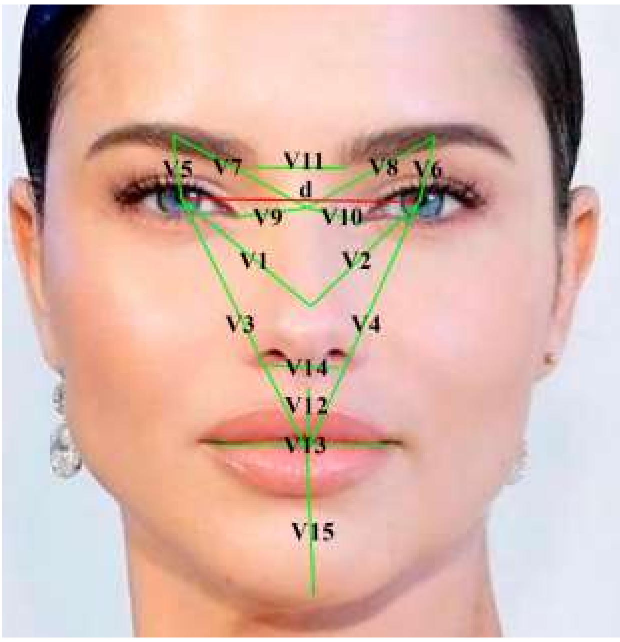

Fig. 1. **Distances between facial landmarks used for feature vector creation**
Source: **compiled by the authors**

After reviewing the referenced studies [24, 25], researchers identified 17 significant facial landmarks. Utilizing the pixel distances between these landmarks (Fig. 1), a 10-feature vector was constructed. Symmetrical distances were averaged, is crucial as human faces exhibit inherent symmetry. This process consolidates information from corresponding points on both sides of the face, enhancing robustness to variations like lighting and facial expressions

To address facial position variations in images and differences in camera-to-face distances, a normalization process is implemented by dividing each feature by the eye-to-eye distance [26, 27]. The description of the obtained vector is shown in Table 1.

Table 1. **Description of the feature vector**

No.	Equation	Description
1	(v1+v2)/2d	Average distance from eyes to nose midpoint
2	(v3+v4)/2d	Average distance from eyes to mouth center
3	(v5+v6)/2d	Average distance from eyes to eyebrow center
4	(v7+v8)/2d	Average distance from eyebrows to nose apex
5	(v9+v10)/2d	Average distance from eye corners to nose apex
6	v11/d	Inter-eyebrow distance
7	v12/d	Nose to mouth midpoint distance
8	v13/d	Mouth corner distance
9	v14/d	Nose edge distance
10	v15/d	Mouth to chin distance

Source: **compiled by the authors**

A dataset sourced from reference [28] comprising 200 photographs for each of the two persons was selected. Out of these, 100 photos were allocated for constructing a prediction ellipsoid to recognize the first person, while the remaining 300 were designated for testing purposes. Consequently, 400 feature vectors, each comprising 10 elements, were acquired, with one vector corresponding to each photo.

The mean vector of samples $\overline{\mathrm{X}} = \{\overline{X}_1, \overline{X}_2, \ldots, \overline{X}_{10}\}^T$ of the first person to build the prediction ellipsoid is represented as: $\overline{\mathrm{X}}$ = {0.6330; 1.1629; 0.3010; 0.6259; 0.2994; 0.3670; 0.3015; 0.8011; 0.3584; 0.6290}.

Additionally, the covariance matrix of the sample is presented in Table 2, while Table 3 delineates the characteristic ranges.

The Mardia test was employed to evaluate the deviation of the multivariate data distribution from normality. This test relies on analyzing the multivariate skewness β_1 and kurtosis β_2 of the dataset, serving as metrics for gauging the extent of deviation from the normal distribution.

These parameters are computed using the following formulas:

$$\beta_1 = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left\{\left(\mathrm{X}_i - \overline{\mathrm{X}}\right)^T \mathrm{S}_{\mathrm{X}}^{-1}\left(\mathrm{X}_j - \overline{\mathrm{X}}\right)\right\}^3, \tag{7}$$

$$\beta_2 = \frac{1}{N}\sum_{j=1}^{N}\left\{\left(\mathrm{X}_j - \overline{\mathrm{X}}\right)^T \mathrm{S}_{\mathrm{X}}^{-1}\left(\mathrm{X}_j - \overline{\mathrm{X}}\right)\right\}^2. \tag{8}$$

As per the Mardia test, the multivariate distribution of the obtained sample is deemed non-Gaussian. This determination is based on the test statistic for multivariate skewness $N\beta_1/6$, which stands at 289.20, surpassing the Chi-Square distribution quantile of 277.77 for 220 degrees of freedom and a significance level of 0.005. Conversely, the test statistic for multivariate kurtosis β_2, measuring 122.35, does not exceed the Gaussian distribution quantile of 127.97. This reference value corresponds to a mean of 120, a variance of 9.6, and a significance level of 0.005. Consequently, there arises a necessity to implement a normalizing transformation (3).

Table 2. **The covariance matrix of the initial sample**

0.00101	0.00089	-0.00060	-0.00006	-0.00012	-0.00011	-0.00052	0.00001	0.00034	-0.00113
0.00089	0.00200	0.00000	0.00026	-0.00016	0.00006	0.00052	-0.00004	0.00046	0.00024
-0.00060	0.00000	0.00111	0.00047	0.00001	0.00019	0.00071	-0.00009	-0.00009	0.00121
-0.00006	0.00026	0.00047	0.00039	-0.00003	0.00017	0.00031	-0.00027	-0.00006	0.00034
-0.00012	-0.00016	0.00001	-0.00003	0.00008	0.00002	0.00003	0.00000	-0.00010	0.00008
-0.00011	0.00006	0.00019	0.00017	0.00002	0.00073	0.00027	0.00003	-0.00002	0.00002
-0.00052	0.00052	0.00071	0.00031	0.00003	0.00027	0.00119	-0.00015	-0.00007	0.00149
0.00001	-0.00004	-0.00009	-0.00027	0.00000	0.00003	-0.00015	0.00276	0.00100	0.00129
0.00034	0.00046	-0.00009	-0.00006	-0.00010	-0.00002	-0.00007	0.00100	0.00104	0.00016
-0.00113	0.00024	0.00121	0.00034	0.00008	0.00002	0.00149	0.00129	0.00016	0.00416

Source: **compiled by the [28]**

Table 3. **Ranges of characteristics of the initial sample**

	1	2	3	4	5	6	7	8	9	10
Min	0.55734	1.07032	0.23998	0.57780	0.27292	0.31088	0.22610	0.69870	0.29332	0.46811
Max	0.70846	1.28726	0.39580	0.66983	0.32167	0.43156	0.39956	1.00075	0.46041	0.80951

Source: **compiled by the [28]**

The Box-Cox transformation is a statistical technique designed to stabilize variance and bring a dataset into closer alignment with the assumptions of normality, especially when dealing with skewed data.

This univariate transformation targets a single variable or univariate data, employing the formula [29]:

$$Z_j = x(\lambda_j) = \begin{cases} \left(X_j^{\lambda_j} - 1\right) / \lambda_j, \lambda_j \neq 0; \\ \ln(X_j), \quad \lambda_j = 0. \end{cases} \quad (9)$$

It is particularly advantageous when addressing issues like heteroscedasticity or non-normality in the dataset. The selection of the optimal value of λ is a crucial step, often accomplished through optimization techniques like maximum likelihood estimation or cross-validation. Once the optimal λ is determined, the transformation is applied to the original data, resulting in a new dataset that better adheres to the assumptions of normality and homoscedasticity.

In the context of the original BCT, a univariate transformation is performed with a single parameter λ, applied element-wise to a vector. In the case of multivariate data, this transformation is typically applied k times as a univariate mapping to each column, each with its unique λ value from k-variate vector $\Theta = \{\lambda_1, \lambda_2, \ldots, \lambda_k\}$. The determination of the optimal λ is a critical step, aiming to bring the distribution of the transformed output value as close as possible to a normal distribution.

Maximum likelihood estimation stands out as a popular method for finding this optimal λ value [30]:

$$l(\lambda) = -\frac{N}{2}\ln\sum_{i=1}^{N}\frac{\left(x(\lambda)_i - \overline{x(\lambda)}\right)^2}{N} + (\lambda - 1)\sum_{i=1}^{N}\ln(x_i)\,. \quad (10)$$

The multivariate BTC extends the concept of the univariate BCT to accommodate multiple variables or multivariate data. The multivariate BTC method employs a separate transformation parameter for each variable in the dataset. This approach allows for tailoring the transformation to the characteristics of each variable individually. When the variables are transformed to achieve joint normality, they tend to exhibit approximate linear relationships, remain constant in conditional variance, and approximate marginal normality in distribution. In the context of using a multivariate BCT, the components of the transformation vector Z are defined as (9).

For the multivariate Box-Cox transformation, the likelihood function is:

$$l(X, \theta) = \sum_{j=1}^{k}(\lambda_j - 1)\sum_{i=1}^{N}\ln(x_{ji}) - \frac{N}{2}\ln\left[\det(S_Z)\right]. \quad (11)$$

Following the implementation of normalizing transformations, a prediction ellipsoid is constructed using (5):

$$\left(Z - \overline{Z}\right)^T S_Z^{-1}\left(Z - \overline{Z}\right) = \chi^2_{10,\,0.005}\,. \quad (12)$$

The critical value of the Chi-square distribution corresponding to a significance level of 0.005 and 10 degrees of freedom is determined to be 25.19.

Based on (6), a ten-variate prediction ellipsoid was constructed utilizing the F-distribution:

$$\left(Z - \overline{Z}\right)^T S_Z^{-1}\left(Z - \overline{Z}\right) = \frac{k\left(N^2 - 1\right)}{N(N - k)} F_{10,\,90,\,0.005}\,. \quad (13)$$

The critical value of the F-distribution corresponding to a significance level of 0.005, 10, and 90 degrees of freedom is determined to be 2.77. The right side of the equation is 30.77.

NORMALIZATION OF THE TRAINING SAMPLE AND CONSTRUCTION OF PREDICTION ELLIPSOIDS

The initial sample undergoes a univariate BCT. Upon solving the task using the maximum likelihood method of the logarithmic function (10), obtained the following parameter estimates: $\hat{\lambda}_1$ = 1.7451, $\hat{\lambda}_2$ = -4.5493, $\hat{\lambda}_3$ = -0.7145, $\hat{\lambda}_4$ = 0.3643, $\hat{\lambda}_5$ = 5.1055, $\hat{\lambda}_6$ = -0.8785, $\hat{\lambda}_7$ = -0.4222, $\hat{\lambda}_8$ = 2.7221, $\hat{\lambda}_9$ = -1.8611, $\hat{\lambda}_{10}$ = 1.0102.

Following the application of the univariate Box-Cox transformation using components (9), where each element of vector Z is computed independently of the others, a sample was derived where the vector of means $\overline{Z} = \{\overline{Z}_1, \overline{Z}_2, \ldots, \overline{Z}_{10}\}^T$ is $\overline{Z}$ = {-0.31461; 0.10718; -1.92555; -0.43103; -0.19545; -1.62004; -1.57614; -0.31836; -3.16345; -0.37015}. Additionally, the covariance matrix of the sample is presented in Table 4, while Table 5 delineates the characteristic ranges.

The normalized sample, achieved through the application of the univariate BCT, does not deviate from the multivariate normal distribution. This is evident as the test statistic for multivariate skewness $N\beta_1/6$ measures 276.51, falling below the critical threshold of 277.77. Similarly, the test statistic for multivariate kurtosis β2 registers at 120.4, remaining below the critical value of 127.97.

The initial sample undergoes the ten-variate BCT. Upon solving the task using the maximum likelihood method of the logarithmic function (11), obtained the following parameter estimates: $\hat{\lambda}_1$ = -0.5799, $\hat{\lambda}_2$ = -0.9732, $\hat{\lambda}_3$ = 0.4871,

$\hat{\lambda}_4 = 2.888$, $\hat{\lambda}_5 = 4.0714$, $\hat{\lambda}_6 = -0.231$, $\hat{\lambda}_7 = 0.087$, $\hat{\lambda}_8 = -1.2973$, $\hat{\lambda}_9 = -1.1866$, $\hat{\lambda}_{10} = 1.3321$.

Utilizing the ten-variate Box-Cox transformation to normalize the initial sample using components (9), the sample was derived where the vector of means $\overline{Z} = \{\overline{Z}_1, \overline{Z}_2, \ldots, \overline{Z}_{10}\}^T$ is $\overline{Z}$ = {-0.31461; 0.10718; -1.92555; -0.43103; -0.19545; -1.62004; -1.57614; -0.31836; -3.16345; -0.37015}. Additionally, the covariance matrix of the sample is presented in Table 6, while Table 7 delineates the characteristic ranges.

Following the normalization of data using both univariate and multivariate Box-Cox transformations, ten-variate ellipsoids were constructed according to using (12) and (13). Subsequently, a computer program was developed to execute experiments based on the constructed models. This program was developed using the Python programming language.

Based on decision rules (1, 2, 12, 13), eight prediction ellipsoids were created for both non-normalized and normalized data using logarithmic transformation [28], univariate, and multivariate Box-Cox transformations with Chi-square and F-distribution quantiles.

A comparative analysis was conducted using a test set consisting of 300 images, where 100 photographs belong to person 1, for which prediction ellipsoids were build, and 200 are photographs of another person.

Table 4. **The covariance matrix of the sample normalized through the univariate Box-Cox transformation (BCT)**

0.00051	0.00027	-0.00329	-0.00006	0.00000	-0.00054	-0.00197	0.00006	0.00437	-0.00080
0.00027	0.00035	-0.00016	0.00014	0.00000	0.00022	0.00115	-0.00006	0.00294	0.00005
-0.00329	-0.00016	0.06670	0.00489	0.00000	0.00923	0.02936	-0.00234	-0.01845	0.00937
-0.00006	0.00014	0.00489	0.00070	0.00000	0.00149	0.00234	-0.00085	-0.00193	0.00046
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	-0.00001	0.00000
-0.00054	0.00022	0.00923	0.00149	0.00000	0.03103	0.00950	0.00025	-0.00533	0.00025
-0.00197	0.00115	0.02936	0.00234	0.00000	0.00950	0.03558	-0.00250	-0.01137	0.00810
0.00006	-0.00006	-0.00234	-0.00085	0.00000	0.00025	-0.00250	0.01315	0.03704	0.00273
0.00437	0.00294	-0.01845	-0.00193	-0.00001	-0.00533	-0.01137	0.03704	0.34189	0.00232
-0.00080	0.00005	0.00937	0.00046	0.00000	0.00025	0.00810	0.00273	0.00232	0.00412

Source: **compiled by the authors**

Table 5. **Ranges of characteristics of the sample normalized through the univariate BCT**

	1	2	3	4	5	6	7	8	9	10
Min	-0.36642	0.05845	-2.48076	-0.49720	-0.19561	-2.03872	-2.06852	-0.60754	-4.72990	-0.53009
Max	-0.25900	0.15013	-1.31438	-0.37285	-0.19527	-1.24335	-1.12041	0.00075	-1.73859	-0.19029

Source: **compiled by the authors**

Table 6. **The covariance matrix of the sample normalized through the ten-variate BCT**

0.00436	0.00134	-0.00233	-0.00006	-0.00001	-0.00080	-0.00323	0.00007	0.00616	-0.00204
0.00134	0.00106	-0.00002	0.00008	0.00000	0.00017	0.00112	-0.00004	0.00281	0.00013
-0.00233	-0.00002	0.00374	0.00036	0.00000	0.00115	0.00381	-0.00035	-0.00198	0.00192
-0.00006	0.00008	0.00036	0.00007	0.00000	0.00023	0.00038	-0.00018	-0.00027	0.00012
-0.00001	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	-0.00002	0.00000
-0.00080	0.00017	0.00115	0.00023	0.00000	0.00846	0.00270	0.00012	-0.00122	0.00009
-0.00323	0.00112	0.00381	0.00038	0.00000	0.00270	0.01047	-0.00090	-0.00277	0.00378
0.00007	-0.00004	-0.00035	-0.00018	0.00000	0.00012	-0.00090	0.00705	0.01390	0.00175
0.00616	0.00281	-0.00198	-0.00027	-0.00002	-0.00122	-0.00277	0.01390	0.08567	0.00116
-0.00204	0.00013	0.00192	0.00012	0.00000	0.00009	0.00378	0.00175	0.00116	0.00305

Source: **compiled by the authors**

Table 7. **Ranges of characteristics of the sample normalized through the ten-variate BCT**

	1	2	3	4	5	6	7	8	9	10
Min	-0.69589	0.06576	-1.02852	-0.27523	-0.24437	-1.34124	-1.39454	-0.45651	-2.76948	-0.47758
Max	-0.38152	0.22388	-0.74583	-0.23742	-0.24319	-0.92748	-0.88171	0.00075	-1.27277	-0.18419

Source: **compiled by the authors**

The final evaluation of the model's quality in face recognition task should consider not only individual metrics but also their alignment with specific requirements and the application context. Currently, there is no universal set of metrics suitable for all scenarios. Therefore, it is recommended to carefully select and combine metrics, considering the task's specificity, to obtain the most comprehensive and relevant assessment of the model's performance.

Three important metrics were used to evaluate the quality of recognition of the created models: the probability of recognizing person 1 (PRP), type II errors, and accuracy.

PRP measures how accurately the model recognizes the face for which the corresponding prediction ellipsoid was built.

Type II errors allow evaluation of the probability with which the model mistakenly identifies another person as the one for whom the prediction ellipsoid was built. For example, if the prediction ellipsoid is created for person A, but the model mistakenly recognizes person B as person A, this is considered a type II error. Minimizing such errors is important for increasing the model's reliability.

Accuracy is a general metric that evaluates the proportion of correct predictions made by the model. To assess accuracy, the ratio of the number of correct predictions to the total number of predictions is used.

The results of applying prediction ellipsoids to non-normalized and normalized data using the logarithm, univariate BCT, and ten-variate BCT with different quantiles of the distributions are shown in Table 8.

As it is evident from Table 8, the best accuracy has the application of prediction ellipsoid for normalized data based on ten-variate BCT with Chi-square quantile.

DISCUSSION AND FUTURE RESEARCH

When utilizing the Chi-square distribution quantile for constructing prediction ellipsoids, a consistent enhancement in accuracy is observed across all normalization techniques compared to non-normalized data. This underscores the pivotal role of normalization in augmenting recognition performance. Notably, the most substantial accuracy improvement, reaching 97.333 %, is achieved with the application of ten-variate BCT normalization. This underscores the effectiveness of multivariate normalization techniques in enhancing recognition accuracy when employing the Chi-square distribution.

In contrast to the Chi-square distribution, the F-distribution quantile yields diverse outcomes across various normalization techniques. While certain techniques exhibit improved accuracy compared to non-normalized data, others display a decrease in accuracy. Intriguingly, the highest PRP of 100% is attained when applying the F-distribution with normalized data. However, for specific normalization techniques, the F-distribution might not yield significant enhancements in recognition accuracy, as the accuracy of application univariate BCT reached the lowest value, even compared with the non-normalized data. The best accuracy for the prediction ellipsoid with the quantile of F-distribution was obtained by applying it to the normalized data using the logarithm and the ten-variate BCT.

Across both distribution quantiles, it is evident that normalization techniques, particularly multivariate methods like the ten-variate BCT, consistently contribute to improved recognition accuracy in comparison to non-normalized data. The selection of the distribution quantile significantly influences the efficacy of normalization techniques on recognition accuracy. While the Chi-square distribution generally leads to improved accuracy, the impact of the F-distribution varies depending on the normalization technique.

The selection of distribution quantiles can vary depending on the field of application. In the entertainment industry, opting for the F-distribution is sensible because prioritizing the recognition of person 1 outweighs the concern for type II errors. Conversely, in security system development, minimizing type II errors is crucial, and this objective is achieved by using the Chi-square quantile.

Table 8. **Comparison of the results**

Quantiles	Metrics	Non-normalized, %	Log, %	Univariate BCT, %	Ten-variate BCT, %
Chi-square	PRP	92	95	94	97
	Type II errors	4.5	5.5	5.5	2.5
	Accuracy	94.333	94.667	94.333	97.333
F-distribution	PRP	99	100	100	100
	Type II errors	6.5	6.5	10	6.5
	Accuracy	95.333	95.667	93.333	95.667

Source: **compiled by the authors**

Overall, the prospect of further research in facial recognition and prediction ellipsoid construction is promising. Applying other multivariate normalization techniques, such as the Johnson transformation, holds promise for advancing research in facial recognition and prediction ellipsoid construction. Furthermore, focusing on the right part of the prediction ellipsoid represents a valuable avenue for future research. This could involve exploring methods for ellipsoid parameters or developing novel approaches for identifying and prioritizing key facial features. By addressing these challenges and exploring new methodologies, it is possible to create more effective and reliable facial recognition systems with broader applications in various domains.

CONCLUSIONS

Normalization enhances recognition accuracy, across both Chi-square and F-distribution quantiles, normalization techniques consistently lead to improved recognition accuracy compared to non-normalized data. This underscores the importance of normalization in enhancing the effectiveness of prediction ellipsoids for facial recognition tasks.

Multivariate normalization outperforms univariate. The results indicate that multivariate normalization techniques, such as the ten-variate Box-Cox transformation, consistently yield better

recognition accuracy compared to univariate normalization methods. This suggests that capturing relationships between multiple facial features enhances the probability of face recognition as a result of application prediction ellipsoids.

Chi-Square distribution outperforms F-distribution. Generally, prediction ellipsoids constructed using the Chi-square distribution quantile exhibit better recognition accuracy compared to those constructed using the F-distribution quantile. This suggests that the Chi-square distribution may be more suitable for facial recognition tasks, especially in scenarios where minimizing type II errors is crucial.

Further investigation is required to understand the underlying factors contributing to the observed differences in recognition accuracy. Future research could explore additional normalization techniques, and model parameters to optimize the recognition probability of prediction ellipsoids for facial recognition tasks.

REFERENCES

1. Peng, P., Portugal, I., Alencar, P. & Cowan, D. “A face recognition software framework based on principal component analysis”. *Plos one*. 2021, 16 (7): e0254965, https://www.scopus.com/record/display.uri?eid=2-s2.0-85111020812&origin=inward&txGid=55f3322d82f887760a7bdfdcbe01271d. DOI: https://doi.org/10.1371/journal.pone.0254965.

2. Kortli, Y., Jridi, M., Al Falou, A. & Atri, M. “Face recognition systems: A survey”. *Sensors*. 2020; 20 (2): 342. DOI: https://doi.org/10.3390/s20020342.

3. Li, C., Shao, Y, Chen, W., Wang, Z. & Deng, N.“Generalized two-dimensional linear discriminant analysis with regularization”. *Neural Networks*. 2021; 142: 73–91. DOI: https://doi.org/10.48550/arXiv.1801.07426.

4. Shanthi, K., Vidhya, S., Vishakha, K., Subiksha, S., Srija, K. & Mamtha, R. “Algorithms for face recognition drones”. *Materials Today: Proceedings*. 2023; 80: 2224–2227. DOI: https://doi.org/10.1016/j.matpr.2021.06.186.

5. Long, H., Kwon, O., Lee, S. & Kwon, K. “Face recognition using correlation filters and support vector machine in machine learning approach”. *Journal of Korea Multimedia Society*. 2021; 24 (4): 528–537. DOI: https://doi.org/10.1142/S0218001402001575.

6. Sugiharti, E. & Putra, A. “Facial recognition using two-dimensional principal component analysis and k-nearest neighbor: a case analysis of facial images”. *Journal of Physics: Conference Series*. 2020; 1567 (3): 032028. DOI: https://doi.org/10.1088/1742-6596/1567/3/032028.

7. Pranav, K. & Manikandan, J. “Design and evaluation of a real-time face recognition system using convolutional neural networks”. *Procedia Computer Science*. 2020; 171: 1651–1659. DOI: https://doi.org/10.1016/j.procs.2020.04.177.

8. Li, L., Mu, X., Li, S. & Peng, H. “A Review of Face Recognition Technology”. *IEEE Access*. 2020; 8: 139110–139120, https://www.scopus.com/record/display.uri?eid=2-s2.0-85089594679&origin=inward&txGid=d003c411674185eb4d89be9275e06f49 DOI: https://doi.org/10.1109/ACCESS.2020.3011028.

9. Bezerra, V., Costa, V., Barbon Junior, S., Miani, R. & Zarpelão, B. "A one-class classification approach to detect botnets in internet of things devices. Sensors (Basel)". *IoTDS*. 2019; 19 (14): 3188. DOI: https://doi.org/10.3390/s19143188.

10. Kim, S., Park, D. & Jung, J. "Evaluation of one-class classifiers for fault detection: Mahalanobis classifiers and the mahalanobis–taguchi system". *Processes*. 2021; 9 (8): 1450. DOI: https://doi.org/10.3390/pr9081450.

11. Vaiyapuri, T. & Binbusayyis, A. "Application of deep autoencoder as an one-class classifier for unsupervised network intrusion detection: a comparative evaluation". *PeerJ Computer Science*. 2020; 6: e327. DOI: https://doi.org/10.7717/peerj-cs.327.

12. Oza, P. & Patel, V. "Active authentication using an autoencoder regularized CNN-based one-class classifier". *14th IEEE International Conference on Automatic Face & Gesture Recognition (FG 2019)*. 2019. pp. 1-8. DOI: https://doi.org/10.1109/FG.2019.8756525.

13. Golestaneh, F., Pinson, P., Azizipanah-Abarghooee, R. & Gooi, H. B. "Ellipsoidal prediction regions for multivariate uncertainty characterization". *IEEE Transactions on Power Systems*. 2018; 33 (4): 4519–4530, https://www.scopus.com/authid/detail.uri?authorId=53163794900. DOI: https://doi.org/10.1109/TPWRS.2018.2791975.

14. Prykhodko, S., Prykhodko, N., Makarova, L., Kudin, O., Smykodub, T. & Prykhodko, A. "Detecting bivariate outliers on the basis of normalizing transformations for non-Gaussian data". *Advanced Information Systems and Technologies, Proceedings of the V International Scientific Conference*. 2017. p. 17–19.

15. Etherington, T. R. "Mahalanobis distances for ecological niche modelling and outlier detection: implications of sample size, error, and bias for selecting and parameterising a multivariate location and scatter method". *PeerJ*. 2021; 9. DOI: https://doi.org/10.7717/peerj.11436.

16. Sun, Y., Jiang, L., Pan, J., Sheng, S. & Hao, L. "A satellite imagery smoke detection framework based on the Mahalanobis distance for early fire identification and positioning". *International Journal of Applied Earth Observation and Geoinformation.* 2023; 118: 103257. DOI: https://doi.org/10.1016/j.jag.2023.103257.

17. Ghorbani, H. "Mahalanobis distance and its application for detecting multivariate outliers". *Facta Universitatis, Series: Mathematics and Informatics*. 2019. p. 583–595. DOI: https://doi.org/10.22190/FUMI1903583G.

18. Khatun, N. "Applications of normality test in statistical analysis". *Open Journal of Statistics*. 2021; 11 (01): 113. DOI: https://doi.org/10.4236/ojs.2021.111006.

19. Prykhodko, S., Makarova, L., Prykhodko, K. & Pukhalevych, A. "Application of transformed prediction ellipsoids for outlier detection in multivariate non-gaussian data". *IEEE 15th International Conference on Advanced Trends in Radioelectronics, Telecommunications and Computer Engineering (TCSET)*. 2020. p. 359–362, https://www.scopus.com/record/display.uri?eid=2-s2.0-85086308714&origin=resultslist&sort=plf-f. DOI: https://doi.org/10.1109/TCSET49122.2020.235454.

20. Prykhodko, S., Prykhodko, A. & Shutko, I. "Estimating the size of web apps created using the cakephp framework by nonlinear regression models with three predictors". *In IEEE 16th International Conference on Computer Sciences and Information Technologies (CSIT)*. 2021; 1: 333–336, http://www.scopus.com/inward/record.url?eid=2-s2.0-85124793057&partnerID=MN8TOARS. DOI: https://doi.org/10.1109/CSIT52700.2021.9648680.

21. McNeish, D. "Should We Use F-Tests for Model Fit Instead of Chi-Square in overidentified structural equation models?". *Organizational Research Methods*. 2020; 23(3): 487–510. DOI: https://doi.org/10.1177/1094428118809495.

22. Khabarlak, K. & Koriashkina, L. "Fast facial landmark detection and applications: A Survey". *Journal of Computer Science and Technology*. 2022; 22 (1): e02. DOI: https://doi.org/10.13140/RG.2.2.32735.07847/1.

23. Haghpanah, M. A., Saeedizade, E., Masouleh, M. T. & Kalhor, A. "Real-time facial expression recognition using facial landmarks and neural networks". *International Conference on Machine Vision and Image Processing (MVIP)*. 2022. p. 1–7. DOI: https://doi.org/10.1109/MVIP53647.2022.9738754.

24. Juhong, A. & Pintavirooj, C. "Face recognition based on facial landmark detection". *10th Biomedical Engineering International Conference (BMEiCON)*. 2017. p. 1–4. DOI: https://doi.org/10.1109/BMEiCON.2017.8229173.

25. Gaber, A., Taher, M. F., Wahed, M. A. & Shalaby, N. M. "SVM classification of facial functions based on facial landmarks and animation Units". *Biomedical Physics & Engineering Express*. 2021; 7 (5): 055008. DOI: https://doi.org/10.1088/2057-1976/ac107c.

26. Wu, Y. & Ji, Q. "Facial landmark detection: A literature survey". *International Journal of Computer Vision*. 2019; 127: 115–142. DOI: https://doi.org/10.1007/s11263-018-1097-z.

27. Johnston, B. & Chazal, P. "A review of image-based automatic facial landmark identification techniques". *J Image Video Proc.* 2018; 86. DOI: https://doi.org/10.1186/s13640-018-0324-4.

28. Prykhodko, S. & Trukhov, A. "Construction of decision rules for face recognition based on the squared Mahalanobis distance for normalized data" (in Ukrainian). *Information Technology: Computer Science, Software Engineering and Cyber Security*. 2023; 2: 50–58, DOI: https://doi.org/10.32782/IT/2023-2-6.

29. Blum, L., Elgendi, M. & Menon, C. "Impact of Box-Cox Transformation on Machine-Learning Algorithms". *Frontiers in Artificial Intelligence*. 2022; 5: 877569. DOI: https://doi.org/10.3389/frai.2022.877569.

30. Atkinson, A. C., Riani, M. & Corbellini, A. "The box–cox transformation: Review and extensions". *Statistical Science*. 2021. p. 239–255. DOI: https://doi.org/10.1214/20-STS778.

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Розпізнавання обличчя за допомогою десятивимірних еліпсоїдів прогнозування для нормалізованих даних з різними квантилями

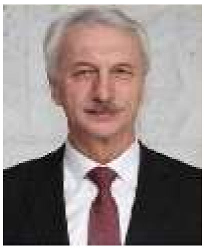
Приходько Сергій Борисович[1), 2)]
ORCID: https://orcid.org/0000-0002-2325-018X; sergiy.prykhodko@nuos.edu.ua. Scopus Author ID: 55225622100
Трухов Артем Сергійович[1)]
ORCID: https://orcid.org/0000-0002-7160-8609; artem.trukhov@gmail.com
[1)] Національний університет кораблебудування імені адмірала Макарова, пр. Героїв України, 9. Миколаїв, 54007, Україна
[2)] Національний університет «Одеська політехніка», пр. Шевченка, 1. Одеса, 65044, Україна

АНОТАЦІЯ

Технологія розпізнавання обличчя відіграє ключову роль у різних сферах, включаючи системи безпеки, розваги та перевірку особи. Однак низька ймовірність ідентифікації людини за обличчям може мати негативні наслідки, що підкреслює необхідність розробки та вдосконалення методів розпізнавання обличчя. Об'єктом дослідження є процес розпізнавання обличчя, предметом дослідження є математична модель розпізнавання обличчя. Одним із поширених підходів у розпізнаванні образів є використання правил прийняття рішень на основі еліпсоїда прогнозування. Значною проблемою в його застосуванні є забезпечення того, щоб дані відповідали багатовимірному нормальному розподілу. Однак дані реального світу часто не відповідають цьому припущенню, що призводить до зниження ймовірності розпізнавання. Таким чином, існує необхідність вдосконалення математичних моделей для врахування таких відхилень. Іншим фактором, який може вплинути на результат, є вибір різних квантилів розподілу, наприклад Хі-квадрат і F-розподілу. Для великих наборів даних використання квантилів Хі-квадрата і F-розподілу в еліпсоїдах прогнозування зазвичай призводить до однакових результатів розпізнавання, але є дані, для яких це не так, і застосування еліпсоїдів прогнозування з різними квантилями розподілів дає різні результати. У цьому дослідженні досліджується застосування еліпсоїдів передбачення в задачах розпізнавання обличчя з використанням різних методів нормалізації та квантилів розподілу. Метою роботи є підвищення ймовірності розпізнавання обличчя шляхом побудови десятивимірного еліпсоїда прогнозування для нормалізованих даних з різними квантилями розподілів. Ми провели експерименти з набором даних зображень обличчя та побудували еліпсоїди прогнозування на основі Хі-квадрат і F-розподілу, використовуючи як одновимірні, так і багатовимірні методи нормалізації. Наші висновки показують, що методи нормалізації значно підвищують точність розпізнавання, при цьому багатовимірні методи, такі як десятивимірне перетворення Бокса-Кокса, перевершують одновимірні підходи. Крім того, еліпсоїди прогнозування, побудовані з використанням квантиля розподілу Хі-квадрат, загалом демонструють кращу ймовірність розпізнавання порівняно з еліпсоїдами, побудованими з використанням квантиля F-розподілу. Подальші дослідження можуть дослідити ефективність застосування інших методів нормалізації, таких як перетворення Джонсона, і проаналізувати побудову еліпсоїдів прогнозування з альтернативними компонентами рівняння еліпсоїда.

Ключові слова: Розпізнавання обличчя; багатовимірний нормальний розподіл; Хі-квадрат; F-розподіл; еліпсоїд прогнозування; нормалізація даних; перетворення Бокса-Кокса

ABOUT THE AUTHORS



Sergiy B. Prykhodko - Doctor of Engineering Science, Professor, Head of Department of Software for Automated Systems. Admiral Makarov National University of Shipbuilding. 9, Heroes of Ukraine Ave. Mykolaiv, 54007, Ukraine
ORCID: https://orcid.org/0000-0002-2325-018X; sergiy.prykhodko@nuos.edu.ua. Scopus Author ID: 55225622100
Research field: Mathematical modeling; computer simulation; stochastic processes; multivariate statistical analysis; system identification; parameter estimation; nonlinear stochastic differential equations; software metrics estimation; pattern recognition

Приходько Сергій Борисович - доктор технічних наук, професор, завідувач кафедри Програмного забезпечення автоматизованих систем Національного університету кораблебудування імені адмірала Макарова, пр. Героїв України, 9. Миколаїв, 54007, Україна

Artem S. Trukhov - postgraduate student at the Department of Software for Automated Systems. Admiral Makarov National University of Shipbuilding. 9, Heroes of Ukraine Ave. Mykolaiv, 54007, Ukraine
ORCID: https://orcid.org/0000-0002-7160-8609; artem.trukhov@gmail.com
Research field: Mathematical modeling; multivariate statistical analysis; parameter estimation; pattern recognition

Трухов Артем Сергійович - аспірант кафедри Програмного забезпечення автоматизованих систем Національного університету кораблебудування імені адмірала Макарова, пр. Героїв України, 9. Миколаїв, 54007, Україна