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Approach to modeling in the metric space of the energy exchange of two media

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ABSTRACT

The work is devoted to the development and analysis of a topological model of the interaction of two energy media in a metric space. The Hausdorff metric space is chosen as the initial set, which obeys the axioms of identity, symmetry and triangle. The real element of the system in the space of energy measures, designed to transfer energy from one medium to another, is represented in correspondence with its cellular image, defined as a virtual Grassmannian. When energy is transferred from a medium with a higher energy to a medium with a lower energy, energy measures determine the organization of processes in the designed heat exchange equipment. Informative components of the Grassmannian are also its area and perimeter. An analysis of the structural model, using the modified Heron formula and the Cayley-Merger determinant, showed that, assuming equilibrium at the Grassmannian nodes, its area in the space of energy measures should be equal to zero. At equilibrium, the semi perimeter in terms of energy measures is the energy potential applied to the element under conditions of its minimum. Relationships between the area of the Grassmannian and the potential applied to the element, the temperature efficiency, and the mixing efficiency of two flows are obtained. The study of this relationship shows that the Grassmannian perimeter has an extremum at an area equal to zero, at which the perimeter is equal to the applied potential. From a design point of view, this means that when specifying flows and inlet temperatures, the requirements for the apparatus are formulated in terms of energy or final temperatures. This essentially determines the required potential and the corresponding energy exchange efficiency. In this case, the potential takes the minimum required value corresponding to the requirement of the project, and the amount of transferred energy corresponds to the required one at fixed values of flows and energy exchange efficiency.

Keywords: Design; energy media interaction; metric space; energy measures; structural model; equilibrium state; heat transfer efficiency

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INTRODUCTION

The design stage is preceded by the statement of the technical task, which is formulated in the form of requirements both for the system and for its constituent elements. As a rule, these requirements are presented in the form of formal conditions at the input and output for elements. In systems, setting these conditions during design is extremely complicated due to the interconnection of elements and requires their expert determination. The above considerations lead to the need to change the modeling paradigm. This is achieved by representing the system by interacting networks (HEN) and, as a result, by determining the conditions at the input and output of the elements based on the requirements of the organization of processes in the system.

To build a new concept for determining the conditions at the input and output in the elements of systems, we will use topological representations. On a Hausdorff space, we introduce a metric manifold with a countable base whose measure is expressed in terms of the difference between the energy measures at the network nodes. The choice of a measure for space is determined by the requirements of classical thermodynamics and satisfies the first and second principles.

LITERATURE REVIEW

Energy problems are significant, since the costs of industry for heating and cooling in the technological cycle reach 50 % of the total consumption [1]. In [2], the optimal control of the process of heat energy exchange is considered, which makes it possible to increase the efficiency of heat exchange equipment. Numerical methods for solving the problems of energy exchange in fluid flows make it possible to optimize, make decisions, and diagnose complex systems [3]. However, such models are based on empirical and correlation [4], which significantly reduces the accuracy of the calculation results. The article [5] proposes an energy balance equation for modeling the energy interactions of liquids with a surface. In [6], the effectiveness of using the theory of fluid-structure interaction is shown, which is confirmed by experiment, which makes it possible to increase the heat transfer coefficient by 16.8-18.5 %. In [7], the problem of modeling an internal flow was considered, including conjugate heat transfer between a liquid and a solid body, which made it possible to obtain good agreement with analytical solutions for direct-flow and counter-flow heat exchangers. The model of an energy transfer network in a porous medium [8], for which local thermal equilibrium cannot be proposed, has shown that it reproduces the effective thermal conductivity in a wide range of liquid and solid thermal conductivity ratios with one set of parameters. The analysis of the structural interaction of the liquid has become an important part of the design process [9] of various industrial components, such as heat exchangers, internal combustion engines. The use of the ANSYS simulation environment has shown the ability to accurately predict fluid behavior and structural interactions numerically. The verification and validation study was carried out using the open source software package OpenFOAM version 6-dev for coupled heat transfer problems [10]. For modeling in [11], transport equations are used, which are solved using a finite element representation based on the method of weighted Galerkin residuals. However, these methods are based on iterative methods and the need to involve additional hypotheses. In [12], an analytical review of heat transfer modeling systems, trends in the development of methods, fundamental limitations, and the need to search for new methods for modeling thermal interaction are presented. Recently, new mathematical methods for analyzing processes have appeared, based on structural modeling and set theory in metric spaces. Thus, in [13], the existence of a g-best point of proximity for a pair of mappings is proved, for which the proposed conjectures about the metric space are used. Some convergence results for the variational inequality problem are established using the variational characteristic of metric projections in a real Hilbert space. The results are applicable to classical problems of optimization theory. The stability of the algorithms is studied and it is shown in [14] that the fixed-point results for various multiplicative contractions are in fact equivalent to the corresponding fixed-point results in metric spaces. The work [15] is devoted to analysis in metric spaces, which has now become an active and mature mathematical discipline, intersecting with a number of other areas. The development of analytical and geometric tools in terms of metric spaces with a measure has led to new applications in potential theory and partial differential equations, metric geometry and other areas. The paper [16] presents the set of all common fixed points of a subfamily of an evolutionary family. For the subset where there are both positive and irrational numbers. In fact, the results obtained from a semigroup of operators are generalized to evolutionary families of operators in a metric space. However, research on the application of structural models and set theory to analyze the interaction of heat flows is not enough. It is worth noting the works [17, 18], [19], which show the possibility of an optimal solution to the problem of designing heat transfer problems by using analytical models. The relevance of changing the concept of designing heat exchange equipment by using the theory of sets is obvious.

PURPOSE AND OBJECTIVES OF THE RESEARCH

The purpose of this work is to substantiate the expediency of analyzing the interaction of the energy of two media using the methods of set theory in a metric temperature space.

To achieve this goal, the following tasks have been set:

1) to develop a topological model of the interaction of energy media in a metric space;

2) to analyze the developed model to determine the efficiency of heat transfer.

TOPOLOGICAL MODEL OF ENERGY EXCHANGE IN TWO MEDIA

To expand the possibilities of thermodynamic methods for analyzing the efficiency of energy transfer, we introduce the logic of geometric methods into consideration. A finite set of integers represented in the form of a graph forms a Hausdorff space [20], in which each integer (node or branch number) can be associated with the value of the energy or flow measure, respectively. In turn, this allows us to define a metric manifold with a countable base that satisfies the following definition [21].

A metric space is a pair (X, d), where X is a set, and d is a numerical function that is defined on a Cartesian product, takes values in the set of non-negative real numbers and is such that

1) $d(x, y) = 0$ if $x = y$ (identity axiom);

2) $d(x, y) = d(y, x)$ (axiom of symmetry);

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3) $d(x,z) \le d(x,y) + d(y,z)$ (triangle axiom).

Wherein

• the set X is called the underlying set of the metric space;

• elements of the set X are called points of the metric space;

• a function d is called a metric

$$d = |x - y|. \quad (1)$$

Here x and y are points in the underlying set X of the metric space of energy measures, for example, the temperature values at the node.

In the space of energy measures, let us put in correspondence with the real element of the system, intended for the transfer of energy from one medium to another, its cellular image, defined in topology as a virtual Grassmannian [22].

In what follows, we will assume that the temperatures at the junction points of the Grassmannian branches are equilibrium. So, for example, the temperature at node No. 1, which combines branches (1-4), (1-2), and (1-3), will be assumed to be equal for all three branches. Similarly for nodes No. 2, No. 3, No. 4.

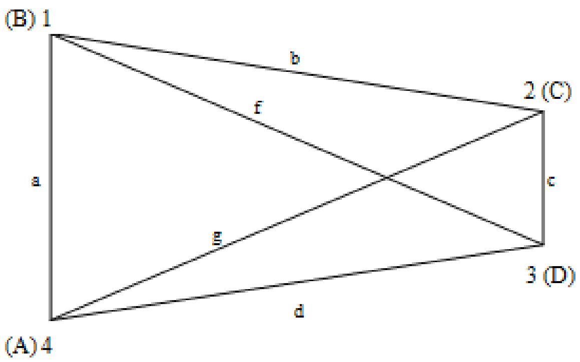

Fig. 1. **Grassmannian in temperature metric space**
Source: **compiled by the authors**

Temperature fields (Fig. 2 and Fig. 3).

Obviously, when energy is transferred from a medium with a higher measure of energy to a medium with a lower measure of energy, the energy measure determines the nature of the ongoing processes and affects the organization of the energy transfer process. However, a qualitative analysis of the temperature fields presented in Fig.1 and Fig. 2 allows us to write a system of inequalities for energy measures at the nodes of a quadrilateral in the space of energy measures one.

For counterflow (Fig. 2)

$$T_1 > T_2 \quad T_4 > T_3. \quad (2)$$

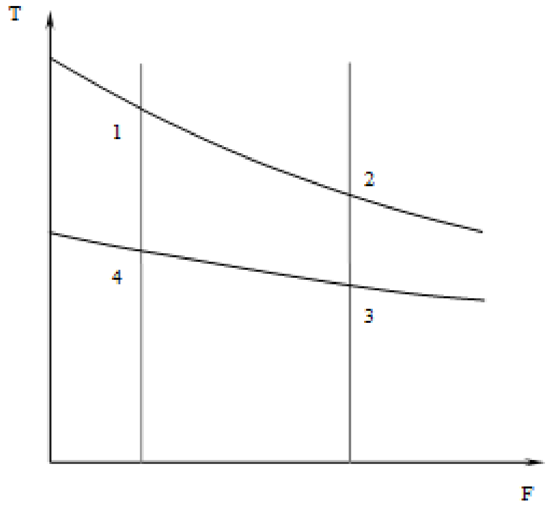

Fig.2. **Counterflow**
Source: **compiled by the authors**

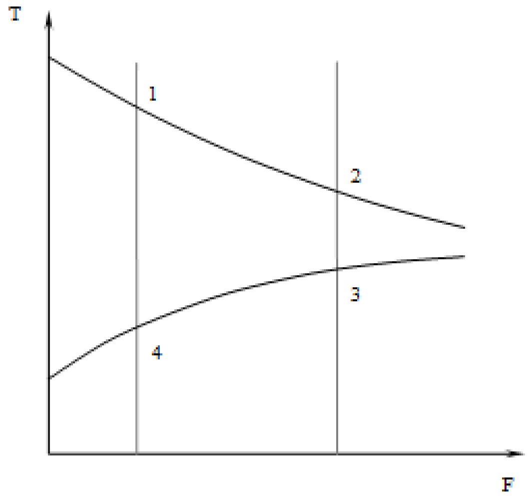

Fig. 3. **Direct flow**
Source: **compiled by the authors**

For direct flow (Fig. 3)

$$T_1 > T_2 \quad T_3 > T_4 \,. \quad (3)$$

In accordance with the introduced concept of space measure (1), the sides of a virtual quadrilateral are defined as

Counter flow Parties	Direct flow Parties
$AB = a = T_1 - T_4$	$AB = a = T_1 - T_4$
$BC = b = T_1 - T_2$	$BC = b = T_1 - T_2$
$CD = c = T_2 - T_3$	$CD = c = T_2 - T_3$
$DA = d = T_4 - T_3$	$DA = d = T_3 - T_4$

Diagonals

$BD = f = T_1 - T_3$

$CA = g = T_2 - T_4$

Diagonals

$BD = f = T_1 - T_3$

$CA = g = T_2 - T_4$.

The model of space measures in the form of a Grassmannian carries information about hot (b) and cold (d) flows, input (a) and output (c) potentials. The area of the figure in the steady state is also informative.

Let's highlight in Fig. 1 are four virtual triangles (A, B, C), (C, D, A) both based on the diagonal g and (D, C, B), (B, A, D) based on the diagonal f. Obviously, the sum of the areas of each pair is equal to the area of the quadrilateral (A, B, C, D).

Let's consider a triangle $(A, B, C) = (a\ b\ g)$. Using the modified Heron formula, we define the area of a triangle as a matrix determinant (the Cayley-Menger determinant [23])

$$-16\,S^2 = \begin{bmatrix} 0 & (T_1 - T_4)^2 & (T_1 - T_2)^2 & 1 \\ (T_1 - T_4)^2 & 0 & (T_2 - T_4)^2 & 1 \\ (T_1 - T_2)^2 & (T_2 - T_4)^2 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}. \quad (4)$$

Because determinant is equal to zero, then we get $S = 0$.

Triangle $(C, D, A) = (c\ d\ g)$.

Just as in the previous case, we get

$$-16\,S^2 = \begin{bmatrix} 0 & (T_2 - T_3)^2 & (T_3 - T_4)^2 & 1 \\ (T_2 - T_3)^2 & 0 & (T_2 - T_4)^2 & 1 \\ (T_3 - T_4)^2 & (T_2 - T_4)^2 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}. \quad (5)$$

Because determinant is equal to zero, then we get $S = 0$.

Triangle $(B, A, D) = (a\ d\ f)$

$$-16\,S^2 = \begin{bmatrix} 0 & (T_1 - T_4)^2 & (T_3 - T_4)^2 & 1 \\ (T_1 - T_4)^2 & 0 & (T_1 - T_3)^2 & 1 \\ (T_3 - T_4)^2 & (T_1 - T_3)^2 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \quad (6)$$

Because determinant is equal to zero, then we get $S = 0$.

Triangle $(B, C, D) = (b\ c\ f)$

$$-16\,S^2 = \begin{bmatrix} 0 & (T_1 - T_2)^2 & (T_2 - T_3)^2 & 1 \\ (T_1 - T_2)^2 & 0 & (T_1 - T_3)^2 & 1 \\ (T_2 - T_3)^2 & (T_1 - T_3)^2 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}. \quad (7)$$

Because determinant is equal to zero, then we get $S = 0$.

ANALYSIS OF THE MAIN FEATURES OF THE TOPOLOGICAL MODEL

From the assumption of equilibrium at the nodes of the Grassmannian, it follows that its area is equal to zero in the space of energy measures.

In addition, from the equality of the areas of triangles to zero, the fulfillment of the axiom of triangles (3) follows in the sense of the equality of the sum of the two sides of the triangles to the third in accordance with the direction of flow:

Counterflow Direct flow

$$\begin{array}{ll} b + c = f & b + g = a \\ b + g = a & f + d = a \\ a + d = f & b + c = f \\ d + g = c & d + c = g \end{array}. \quad (8)$$

Unlike a real element, the fact of energy transfer in its topological representation is reflected by deformation of the sides and, as a result, a change in the hypothetical shape shown in Fig. 1.

If the equilibrium at the nodes is satisfied, then the perimeter of the Grassmannian in terms of energy measures can be easily determined depending on the direction of flow by the following relations:

for counterflow

$$2p = a + b + c + d = 2(T_1 - T_3)\,, \quad (9)$$

p is the semi perimeter,

for direct flow

$$2p = a + b + c + d = 2(T_1 - T_4)\,. \quad (10)$$

It is easy to see that half of the Grassmannian perimeter (p) is the energy potential applied to the element. When the conditions of equilibrium at the nodes are met, the area of the figure during the transfer of energy from one medium to another in a stationary state is equal to zero. In turn, the minimality of the perimeter at S =0 follows from geometric representations and, as a consequence, the minimality of the energy potential applied to the element.

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In turn, this allows us to consider the area of the Grassmannian as a measure of a metric subspace G in the space of energy measures (1).

From the assumption of equilibrium at the nodes in the space of energy measures, three zero identities follow, determined by the direction of movement of the interacting flows:
counterflow

$$b+d-(f-g)=0 \tag{11}$$

$$b-d-(a-c)=0 \tag{12}$$

$$g+f-(a+c)=0, \tag{13}$$

direct flow

$$b+d-(f-g)=0. \tag{14}$$

$$b-d-(a-c)=0 \tag{15}$$

$$g+f-(a+c)=0. \tag{16}$$

Relations (11)-(16) can be considered as equilibrium conditions for the deformation of energy measures in the Grassmannian when it is transferred from a medium with a large energy potential to a medium with a lower energy potential.

As mentioned above, a change in the shape of the Grassmannian due to deformation of the sides can lead to a change in its area. In the accepted notation, the Brahmagupta formula for determining the area of a quadrilateral in the constructed metric space (1), as a function of the semi perimeter, can be written as

$$S=\sqrt{(a-p)\cdot(b-p)\cdot(c-p)\cdot(d-p)-\frac{1}{4}[(a\cdot c+b\cdot d+f\cdot g)\cdot(a\cdot c+b\cdot d-f\cdot g)]}. \tag{17}$$

Complementing relations (11)-(16) with the hypothesis of proportionality between the potential applied to the element and the energy measure adopted by it:

$$b-\Phi\Pi=0, \tag{18}$$

where: Φ is the temperature efficiency; Π is the energy potential at the input to the system elements, and energy balance

$$Yb-d=0, \tag{19}$$

where Y is the mixing efficiency of two streams.

Based on the assumption that the energy measures at the nodes are in equilibrium, we construct systems of equations that determine the energy measures on the branches of the Grassmannians in the case of energy exchange both in counterflow and in direct flow.

Direct flow

$$\begin{pmatrix}-1 & 0 & 0 & \Phi & 0 & 0\\ Y & -1 & 0 & 0 & 0 & 0\\ 1 & 1 & 1 & 1 & 0 & 0\\ 1 & 1 & 1 & -1 & 0 & 0\\ 1 & -1 & 0 & 0 & 1 & -1\\ 0 & 0 & -1 & -1 & 1 & 1\end{pmatrix}\cdot\begin{pmatrix}b\\ d\\ c\\ a\\ g\\ f\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 2p\\ 0\\ 0\\ 0\end{pmatrix}$$

$$\begin{pmatrix}b\\ d\\ c\\ a\\ g\\ f\end{pmatrix}=\begin{pmatrix}\Phi p\\ Y\Phi p\\ -p(\Phi+Y\Phi-1)\\ p\\ -p(\Phi-1)\\ -p(Y\Phi-1)\end{pmatrix}=\begin{pmatrix}\Pi\Phi\\ Y\Pi\Phi\\ -\Pi(\Phi+Y\Phi-1)\\ \Pi\\ -\Pi(\Phi-1)\\ -\Pi(Y\Phi-1)\end{pmatrix}$$

Counterflow

$$\begin{pmatrix}-1 & 0 & 0 & 0 & 0 & \Phi\\ Y & -1 & 0 & 0 & 0 & 0\\ 1 & 1 & 1 & 1 & 0 & 0\\ 1 & 1 & 0 & 0 & 1 & -1\\ 1 & -1 & 1 & -1 & 0 & 0\\ 0 & 0 & -1 & -1 & 1 & 1\end{pmatrix}\cdot\begin{pmatrix}b\\ d\\ c\\ a\\ g\\ f\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 2p\\ 0\\ 0\\ 0\end{pmatrix}$$

$$\begin{pmatrix}b\\ d\\ c\\ a\\ g\\ f\end{pmatrix}=\begin{pmatrix}\Phi p\\ Y\Phi p\\ -p(\Phi-1)\\ -p(Y\Phi-1)\\ -p(\Phi+Y\Phi-1)\\ p\end{pmatrix}=\begin{pmatrix}\Pi\Phi\\ Y\Pi\Phi\\ -\Pi(\Phi-1)\\ -\Pi(Y\Phi-1)\\ -\Pi(\Phi+Y\Phi-1)\\ \Pi\end{pmatrix}.$$

The variable introduced into consideration determines the efficiency when mixing both streams

$$Y = \frac{U}{V} = \frac{1-Z}{Z} \qquad \frac{1}{Y+1} = Z = \Phi_m, \qquad (20)$$

where: U is the heat capacity of the cold flow; V is the heat capacity of the hot flow.

The Brahmagupta formula (17) for determining the area after substituting energy measures, taking into account the solutions obtained, will be rewritten as:

Counterflow

$$S^2 = -p(\Pi - p)(Y\Pi^2\Phi - Y^2\Pi^2\Phi^2 - \Pi^2\Phi^2 + \\ + \Pi^2\Phi - \Pi p + p^2). \quad (21)$$

Direct flow

$$S^2 = -(\Pi - p)(p - \Pi\Phi)(Y\Pi^2\Phi - Y\Pi^2\Phi^2 - \\ - Y^2\Pi^2\Phi^2 + \Pi\Phi p - \Pi p + p^2). \quad (22)$$

Equations (21) and (22) are equations of the fourth degree with respect to the half-perimeter and allow, at $S = 0$, to obtain a solution in the form

$$p = \begin{pmatrix} \Pi \\ 0 \\ \dfrac{\Pi\left(Z - \sqrt{8Z^2\Phi^2 + Z^2 - 8Z\Phi^2 - 4Z\Phi + 4\Phi^2}\right)}{2Z} \\ \dfrac{\Pi\left(Z + \sqrt{8Z^2\Phi^2 + Z^2 - 8Z\Phi^2 - 4Z\Phi + 4\Phi^2}\right)}{2Z} \end{pmatrix}. \quad (23)$$

From (21) and (22) it follows that the perimeter of the Grassmannian experiences an extremum at $S = 0$. Since the second derivative of the perimeter is positive, then at $S = 0$ it undergoes a minimum, which followed from geometric considerations. And at the same time (23) the perimeter is equal to the applied potential ($p = \Pi$).

As shown above, when the measures of energy at the nodes are in equilibrium, the areas of triangles are equal to zero.

In turn, it follows from this that the expressions for the determinants can be considered as a system of equations with respect to the diagonals of the quadrilateral:

relatively f

$$\begin{matrix} f^4 + \left[-(b+c)^2 - (b-c)^2\right] f^2 + (b-c)^2(b+c)^2 \\ f^4 + \left[-(a+d)^2 - (a-d)^2\right] f^2 + (a-d)^2(a+d)^2 \end{matrix} \quad (24)$$

relatively g

$$\begin{matrix} \left[(c+d)^2 + (c-d)^2\right] g^2 - g^4 - (c-d)^2(c+d)^2 \\ \left[-(a+b)^2 - (a-b)^2\right] g^2 + g^4 + (a-b)^2(a+b)^2 \end{matrix}. \quad (25)$$

The only root that satisfies the requirements for equations (24) $f = f$ and equations (25) $g = g$ is determined by the equilibrium conditions (11)-(16).

Counter flow

$$b + c = a + d = p = \Pi. \qquad (26)$$

The only root that satisfies the requirements for equations (24) $a = a$ and equations (25) $c = c$ is determined by the equilibrium conditions (11)-(16).

Direct flow

$$b + g = d + f = p = \Pi. \qquad (27)$$

DISCUSSION OF THE RESULTS OF THE ANALYSIS

When the equilibrium conditions are met at the nodes and S = 0, the quadrilateral is folded, turning into two segments folded on top of each other. In such a state, the energy measures in the topological image of the real apparatus are in equilibrium in the same way as in the real apparatus at the corresponding nodes.

In the generally accepted scheme of design devices of this type, it is customary to set the inlet flows and temperatures and the requirements for the device in the form of an amount of energy or final temperatures. This essentially determines the required potential and the corresponding efficiency of energy exchange, for example, as follows

$$\Phi = \frac{T_1 - T_2}{p} \qquad \Pi = \frac{Q}{U\,\Phi}. \qquad (28)$$

In this case, the value of the potential takes on the minimum required value corresponding to the requirement of the project, and the amount of transferred energy corresponds to the required value at fixed values of flows and energy exchange efficiency.

CONCLUSIONS

1. A topological model of energy exchange between two media has been developed, which is based on the equilibrium of temperatures at the nodes of the Grassmannian constructed in the Hausdorff space. The introduction of the space metric made it possible to identify additional information features related to the potential and equilibrium of the energy exchange system between two media.

2. The topological representation provides a mathematical relationship between the temperature difference at the inlet through the thermal efficiency and the efficiency of mixing flows at the outlet. In turn, this makes it possible not only to formalize the terms of reference for the development of means for the exchange of energy between two media, but also to identify shortcomings in the formulation of the problem.

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Підхід до моделювання у метричному просторі обміну енергією двох середовищ

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АНОТАЦІЯ

Робота присвячена розробці і аналізу топологічної моделі взаємодії двох енергетичних серед у метричному просторі. У якості первинної множини вибрано хаусдорфовий метричний простір, який підпорядкується аксіомам тотожності, симетрії і трикутника. Реальному елементу системи у просторі мір енергії, призначеному для передачі енергії від одної середи у іншу, який представлено у відповідності її клітковий образ, що визначається як грассманіан. При передачі енергії від середи з більшою енергією до середи з меншою енергією, міри енергії визначають організацію процесів у теплообмінній апаратурі, що проектується. Інформативними складовими грассманіану є також його площа і периметр. Аналіз структурної моделі, з використанням модифікованої формули Герона та визначника Келі-Мергера, показав, що у припущенні рівноваги у вузлах грассманіану наслідком є рівність нулю його площі у просторі мір енергії. При рівновазі периметр у термінах заходів енергії є енергопотенціал, що прикладено до елемента в умовах його мінімальності. Отримано співвідношення зв'язку площі грассманіана з прикладеним до елемента потенціалом, температурною ефективністю та ефективністю змішування двох потоків. Дослідження цього зв'язку показує, що периметр грассманіану має екстремум при площі, що дорівнює нулю, при якій периметр дорівнює прикладеному потенціалу. З точки зору проектування це означає, що при заданні потоків і температури на вході вимоги до апарату формуються у вигляді кількості енергії або кінцевих температур, що по суті визначає необхідний потенціал та відповідну йому ефективність енергообміну. При цьому потенціал набуває мінімально необхідного значення, що відповідає вимогам проекту, а кількість переданої енергії відповідає необхідному при фіксованих значеннях потоків та ефективності енергообміну.

Ключові слова: проектування; взаємодія енергетичних середовищ; метричний простір; міри енергії; структурна модель; рівноважний стан; ефективність теплообміну.

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